Larkin-Imry-Ma state of $^3$He-A in aerogel

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Aerogels: stiff foams composed of up to 99.8% air
Silica aerogel is the world's lowest-density solid: 1 mg/cm³

2.38 g piece of aerogel supports a 2.5 kg brick.

Aerogels hold 15 different records for material properties, including best insulator
Impurity suppression of unconventional Superfluidity

Unconventional Superfluids:
- heavy fermions (UPt3),
- ruthenates, ...,
- Cuprates
- 3He

Disorder can be introduced in superfluid 3He by high porosity silica aerogel
doubly anisotropic superfluid liquid $^3$He-A

$A_{\mu i} = \Delta e^{i\varphi} d_{\mu} (m_i + in_i)$

$p_{x} + ip_{y}$ superconductor

$\text{Sr}_2\text{RuO}_4$

$l = m \times n$
unit vector in orbital space

chiral orbital ferromagnet

anisotropic superfluid density

$\mathbf{l}$ - unit vector in spin space

spin nematic

$\mathbf{d}$ - unit vector in spin space

anisotropic magnetic susceptibility

$1/2 \rho_s (\mathbf{v}_s \cdot \mathbf{l})^2 + 1/2 \rho_{s\perp} (\mathbf{v}_s \times \mathbf{l})^2$

$1/2 \chi_{||} (\mathbf{H} \cdot \mathbf{d})^2 + 1/2 \chi_{\perp} (\mathbf{H} \times \mathbf{d})^2$
two main problems for $^3$He in aerogel

nature of state:
- Vortex liquid?
- Robust phase?
- Larkin-Imry-Ma?
- Quasi long-range order?

life in between:
- Griffiths phase?
- Pseudo-gap state?
- Vortex liquid?

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The diagram shows the phase diagram for $^3$He in aerogel. The phase transitions are indicated by $T_{ca}$ (superfluid transition temperature in aerogel) and $T_{cb}$ (superfluid transition temperature in bulk 3He). The figures $x$, $y$, and $z$ are not clearly labeled in the diagram.
Superfluid coherence length:
$\xi_0 \approx 20 \text{ to } 80 \text{ nm (34 to 0 bar)}$

Silica particle size diameter,
$\delta \approx 3 \text{ nm}$;

distance between strands
$\xi_a \sim 20 \text{ nm}$
is of order of correlation length
strands provide
local random anisotropy!

Larkin-Imry-Ma length, at which
long-range order of $l$ is destroyed
by local random anisotropy

$L \sim \xi_a \xi_0^2 / \delta^2 \sim 1 \mu\text{m}$

(dipole) length of spin-orbit coupling
$E_{\text{so}} = -g_D (l \cdot d)^2$
$\xi_{\text{so}} \sim 10 \mu\text{m}$

$\delta \ll \xi_a \sim \xi_0 \ll L < \xi_{\text{so}}$

length scales in aerogel
Larkin-Imry-Ma effect: collective action of aerogel strings destroys orientational long-range order.

L ~ $\xi_a \left( \frac{\xi_0^2}{\delta^2} \right) \sim 1 \, \mu m$

L ~ $\gg \xi_a \sim 20 \, nm$

Larkin, JETP 31, 784 (1970) destruction of translational order (melting of vortex lattice by impurities)

Imry & Ma, PRL 35, 1399 (1975) destruction of orientational order by random anisotropy
Larkin-Imry-Ma effect
*model for strands: randomly oriented cylinders*

**E(l,n) = E_a (anisotropy) \left[ (l \cdot n)^2 - \frac{1}{3} \right]**

If diameter is smaller than coherence length $\delta \ll \xi_0$

*D. Rainer & M. Vuorio, J.Phys. 10 (1977) 3093*

$$E_a \sim \sigma k_F^2 \left( \Delta^2 / T_c \right) > 0$$

$$\sigma = \xi_a \delta$$

is scattering cross-section of a cylinder

If diameter is smaller than coherence length $\delta \ll \xi_0$
Larkin-Imry-Ma effect:

collective effect of fluctuations of random anisotropy of strands

fluctuation of energy of random anisotropy

( finite size effect in the box $L \times L \times L$)

$$$<E(N)> = 0 ; \text{ variance } = <E^2> = N E_a^2$$$

$E$ as function of orientation of $l$

vector $l$ prefers orientation mimimizing energy

$$E_{\text{min}}(N) = -<E^2(N)>^{1/2} = -N^{1/2} E_a$$

$N = L^3 / \xi_a^3$

total number of cylinders of length $\xi_a$
in volume $L^3$

$$E_a \sim \xi_a \delta k_F^2 (\Delta^2 / T_c)$$
orientational energy of cylinder of length $\xi_a$
estimation of Larkin-Imry-Ma length

in the neighboring boxes $l$ prefers different orientations, this is opposed by gradient energy

competition between orientational energy of fluctuation and gradient energy

energy density of random anisotropy

$$E_{ran} \sim - \langle E^2(N) \rangle^{1/2} L^{-3} \sim - E_a \xi_a^{-3/2} L^{-3/2}$$

density of opposing gradient energy

$$E_{gr} \sim K (\partial l)^2 \sim (k_F^3/m)(\Delta / T_c)^2 L^{-2}$$

$E_{random}(L) + E_{gradient}(L)$ has minimum at Larkin-Imry-Ma length

$$L_{LM} \sim \xi_a (\xi_0^2/\delta^2) \sim 1 \mu m$$
small deformation of aerogel destroys subtle LIM effect & leads to global orientation of $^3\text{He-A}$

Kunimatsu et al.
JETP Lett. 86 (2007), cond-mat/0612007 courtesy of Bunkov
regular anisotropy in deformed aerogel

uniaxial deformation → regular uniaxial anisotropy

\[ E_{reg\ an} = \langle E(l,n) \rangle \sim \frac{\Delta l}{l} E_a \xi_a^{-3} (l \cdot z)^2 \]

\[ E(l,n) = E_a \left[ (l \cdot n)^2 - \frac{1}{3} \right] \]

squeezing: \( \Delta l < 0 \) → easy plane for \( n \) → easy axis for \( l \)

stretching: \( \Delta l > 0 \) → easy axis for \( n \) → easy plane for \( l \)
regular anisotropy may destroy Imry-Ma effect

Energy density of uniaxial anisotropy

$$E_{\text{regular}} \sim \frac{\Delta l}{l} E_a \xi_a^{-3}$$

Energy density of random anisotropy

$$E_{\text{random}} \sim E_a \xi_a^{-3/2} L^{-3/2}$$

Larkin-Imry-Ma state is destroyed by small deformation:

$$E_{\text{regular}} > E_{\text{random}} \quad \text{when} \quad \frac{\Delta l}{l} > (\frac{\xi_a}{L})^{3/2} \sim \frac{\delta^3}{\xi_0^3} \sim 10^{-3} - 10^{-2}$$

3 nm  20 nm  20-80 nm  1 µm  10 µm

δ  <<  ξa  ~  ξ0  <<  L  <  ξso
NMR probe of Larkin-Imry-Ma state

Dmitriev et al. JETP Lett. 84, 461 (2006)

180° pulse removes f-line

cooling through \( T_c \) gives 2 lines in NMR spectrum

\( c \) (close) & \( f \) (far)
NMR lines: in mixed f+c state and in pure c state

Dmitriev et al.

T = 0.77 T_c,
P = 29.3 bar
temperature dependence of NMR frequency shift

Dmitriev et al.

P=29.3 бар

- f-line in mixed f+c state
- c-line in mixed f+c state
- c-line in pure c state
dependence of frequency shift on tipping angle of precession

\[ \frac{(\omega - \omega_L)}{2\pi} = A \cos \beta \]

\[ A = 37.4 \text{ Гц} \]

P = 29.3 бар,
T = 0.76T_{ca}

Dmitriev et al.
dependence of frequency shift on tipping angle of precession

\[ 1 + \cos \beta \]

Dmitriev et al.

\[ P = 29.3 \text{ bar} \]
\[ T = 0.76T_{ca} \]
some samples exhibit large negative frequency shift

Dmitriev et al.

large negative frequency shift

small positive frequency shift

H=284 Э, P=25.5 бар

H=142 Э, P=26.0 бар
large negative frequency shift in deformed aerogel

Bunkov, Kunimatsu, et al.
non-deformed vs deformed aerogel

\[ \frac{\Delta \omega_{c-line}}{|\Delta \omega_{neg}|} \sim 0.03 \]

negative frequency shift in squeezed aerogel reaches −1 kHz
small frequency shift from Larkin-Imry-Ma effect

\[
\frac{\Delta \omega_{c-line}}{|\Delta \omega_{\text{neg}}|} \approx 0.03
\]

\[\Delta \omega = \Delta \omega_{\text{max}} \left[ (l \cdot d)^2 - (l \cdot H)^2 / H^2 \right] \]

dependence of frequency shift \( \Delta \omega \) on orientation of \( l \) with respect to magnetic field \( H \) and vector \( d \)

state with c-line:
small positive shift in disordered Larkin-Imry-Ma state

for random \( l \) and fixed \( d \perp H \)
\[\Delta \omega = 0\]

if \( l \) along \( H \)
\[\Delta \omega = - \Delta \omega_{\text{max}}\]

state with negative shift:
Larkin-Imry-Ma state is destroyed by deformation of aerogel
non-zero NMR shift in random $l$ texture is due to spin-orbit interaction: another regular anisotropy

$$E_{so} = -g_D(l \cdot d)^2$$

spin-orbit coupling slightly aligns $l$ & $d$ by amount

$$E_{so} / E_{random} \sim (L / \xi_{so})^2$$

leading to small frequency shift

$$\frac{\Delta \omega_{c-line}^{\text{theor}}}{|\Delta \omega_{neg}|} \sim (L / \xi_{so})^2$$

$$\frac{\Delta \omega_{c-line}^{\exp}}{|\Delta \omega_{neg}|} \sim 0.03$$

$$\left(\frac{L}{\xi_{so}}\right)^2 \sim 0.03$$

$3 \text{ nm} \quad 20 \text{ nm} \quad 20-80 \text{ nm} \quad 1-3 \mu m \quad 10 \mu m$

$\delta \ll \xi_a \sim \xi_0 \ll L \ll \xi_{so}$
dependence of frequency shift on tipping angle $\beta$ in disordered states

$$\frac{\Delta \omega(\beta)}{|\Delta \omega_{\text{neg}}|} = b \cos \beta + a \left(1 + \cos \beta\right)$$

\[ b = (1/2)(1 - 3< l_z^2 >) \]

\[ a = b = 0 \text{ in full randomness, for finite } L / \xi_{\text{so}} : \]
\[ a, b \sim (L / \xi_{\text{so}})^2 \ll 1 \]

\[ a = (1/6)(1 - 2< \sin^2 \Phi >) \]

\( \Phi \) - angle between \( l_\perp \) and \( d_\perp \) in transverse plane

interpretation of c-state: \( b \cos \beta \)

interpretation of f-state: \( a \left(1 + \cos \beta\right) \)

\( l_\perp \) random
\( d_\perp H \) regular
\( (a = 0) \), uniaxial anisotropy due to\( H \)
\( (b > 0) \)

\( l \) random
\( (b = 0) \),
\( d_\perp \) and \( l_\perp \) are not independent
\( (a > 0) \)
Superfluid density in Larkin-Imry-Ma state

4-th sound measurements

\[ \frac{\rho_{sA}}{\rho_{sB}} \text{ in aerogel} < \frac{\rho_{sA}}{\rho_{sB}} \text{ in bulk} \]

suppression of \( \rho_{sA} \) in LIM state?
Larkin-Imry-Ma state as vortex state

$l$-texture produces continuous vorticity:

$$\nabla \times \mathbf{v}_s = (h/8\pi m)e_{ijk}l_i \nabla j \times \nabla k$$

Continuous vortex - skyrmion

in bulk $^3\text{He-A}$

$$\int \mathbf{v}_s \cdot d\mathbf{r} = (h/m)(e_{ijk}/8\pi) \int dS \cdot (l_i \nabla j \times \nabla k) = h/m N_2$$

$N_2$ - topological charge of skyrmion

Doubly quantized continuous vortex

Position of vortex peak determines the type of vortex.

Intensity of vortex peak determines number of vortices.

NMR Absorption

Experimental observation by NMR in bulk $^3\text{He-A}$ (Helsinki)
Topology of continuous vortex - skyrmion

classes of mapping $S^2 \rightarrow S^2$

disk with one point at infinity

unit sphere of $l$-vector

$l$ - field:

mapping

$\gamma \rightarrow \Gamma$

$\sigma \rightarrow \Sigma$

$N_2 = \frac{1}{8\pi} \epsilon^{ijk} \int dS_i \ l \cdot (\partial_j l \times \partial_k l) = 1$

topological invariant for skyrmion

homotopy group $
\pi_2 = \mathbb{Z}$

$3\text{He-A}$

$N = 2N_2$

circulation

2-component BEC

$N = N_2$
Larkin-Imry-Ma state as vortex state:
random distribution of continuous vortices with core size of order LIM length $L$

topological charge in the cross-section $R \times R$
of the sample

$$N_2 = \frac{1}{8\pi} e^{ijk} \int_{R \times R} dS_i l \cdot (\partial_j l \times \partial_k l)$$

number of vortex lines crossing the cross-section
$$N = 2N_2$$

area law for $< N^2 >$:
LIM state of superfluid $^3$He-A looks similar to the state above Berezinskii-Kosterlitz-Thouless transition
area law for $<N^2>$:
LIM state of superfluid $^3$He-A looks similar to vortex plasma state above Berezinskii-Kosterlitz-Thouless superfluid transition

$\rho_{SA} = 0$ if vortices can freely move
$\rho_{SA} > 0$ if some pinning of vortices occurs

for strong pinning:
$\frac{\rho_{SA}}{\rho_{SB}}$ in aerogel = $\frac{\rho_{SA}}{\rho_{SB}}$ in bulk

in general:
$\frac{\rho_{SA}}{\rho_{SB}}$ in aerogel < $\frac{\rho_{SA}}{\rho_{SB}}$ in bulk

problems:
calculate $\rho_s$ in Larkin-Imry-Ma state
measure $\rho_s$ in c-state
measure $\rho_s$ in f-state
measure $\rho_s$ in uniform states with $l \perp v_S$ & $l \parallel v_S$
Bradley et al. anisotropy of superfluid density

**PHYSICAL REVIEW LETTERS**

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The inferred superfluid fraction as a function of velocity at a temperature of $\sim 0.18 T_c^{\text{bulk}}$. 

For random $l$ texture: 

$$\langle \rho_s \rangle = (1/3) \rho_s || + (2/3) \rho_s \perp$$

$$\rho_s || < \langle \rho_s \rangle < \rho_s \perp$$

agrees with pinned $l$ texture

Estimation of LIM length $L$ from critical velocity orienting $l$: 

$$v_c \sim 1 \text{mm/s} \sim \hat{n}/mL$$

$L \sim 10 \mu m$

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**FIG. 2.** The inferred superfluid fraction as a function of velocity at a temperature of $\sim 0.18 T_c^{\text{bulk}}$. 

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$\rho_s A \perp v_s$

$1/2 (\rho_s || - \rho_s \perp ) (v_s \cdot l)^2$

$\rho_s || < \rho_s \perp$

$\rho_s A \| v_s$

$\rho_s A \perp v_s$

$\rho_s A \| v_s$

$\rho_s A \perp v_s$

$\rho_s A \| v_s$

$\rho_s A \perp v_s$

$\rho_s A \| v_s$

$\rho_s A \perp v_s$

$\rho_s A \| v_s$
conclusion:

* \(^3\)He-A in aerogel loses long-range orientational order
* long-range orientational order is restored by deformation of aerogel
* c-state is most probably Larkin-Imry-Ma state

problems:

* measure critical deformation
* identify f-state
* identify half-quantum vortices
  (they are stable if \( l \parallel H \))
* measure superfluid density \( \rho_s \)
  (in Larkin-Imry-Ma state, in c-state, in f-state, in uniform states)

theoretical

* quasi long-range order:
  Emig, Bogner & Nattermann, PRL 83, 400 (1999)
  "Nonuniversal Quasi-Long-Range Order in the Glassy Phase of Impure Superconductors"
* long-range correlation of silicon strands:
* role of topological defects
  (hedgehogs, singular vortices, solitons, skyrmions)
* NMR spectrum in all these states
**Half-quantum vortex**

(Alice string)

$N_1 = 1/2$

$\Phi_0 = \frac{hc}{e}$

fractional magnetic flux in electrically charge condensate

$d$ - field:

$d \sim e^{i\Phi} \sigma_{\alpha\beta} d^k (e_1 + ie_2)$

$d$ - unit vector in spin space $SO(3)$

$e_1, e_2, l = e_1 \times e_2$ - triad in orbital space $SO(3)$

$SO(3) \Rightarrow U(1)$

$SO(3) \times U(1) \Rightarrow U(1)$

$G = SO(3) \times SO(3) \times U(1) \Rightarrow H = U(1) \times U(1) \times \mathbb{Z}_2$

$
\mathbb{Z}_2$ symmetry: after circling $d \Rightarrow -d$ but $\Delta(r)$ must be single valued

this requires $\Phi \Rightarrow \Phi + \pi$ after circling

$\Phi = \frac{\phi}{2}$ half-quantum vortex
Alice string in spinor BEC

\[ \Psi(r) = \mathbf{d}(r)e^{i\phi} \]

- positron
- electron

Particle continuously transforms to antiparticle after circling around the Alice string

Alice string in GUT (A. Schwarz)

Person traveling around string can annihilate with person who was at home