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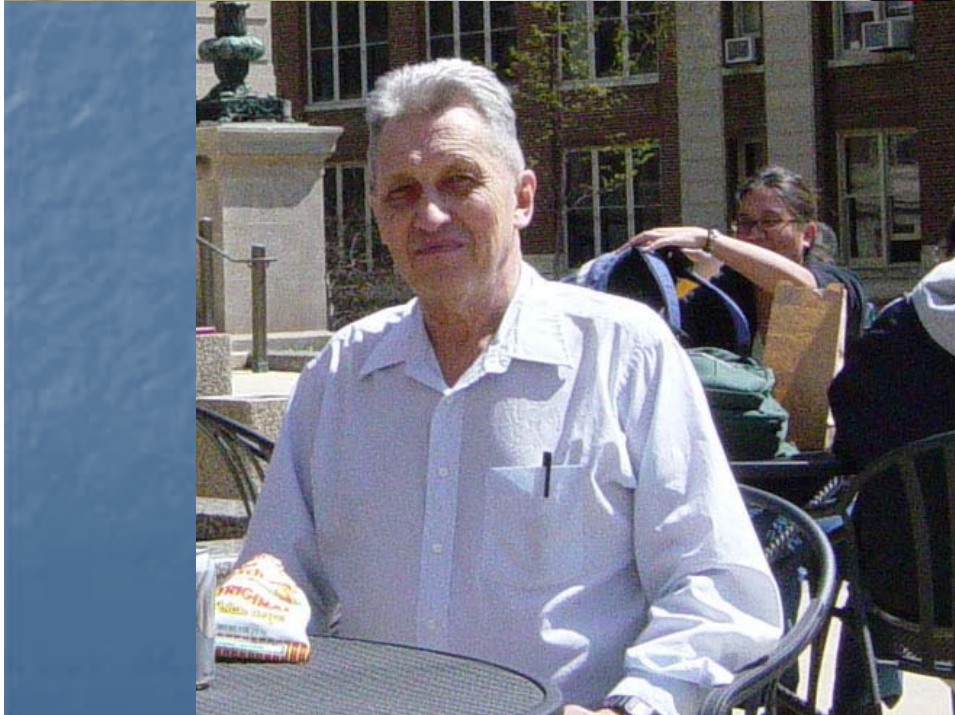
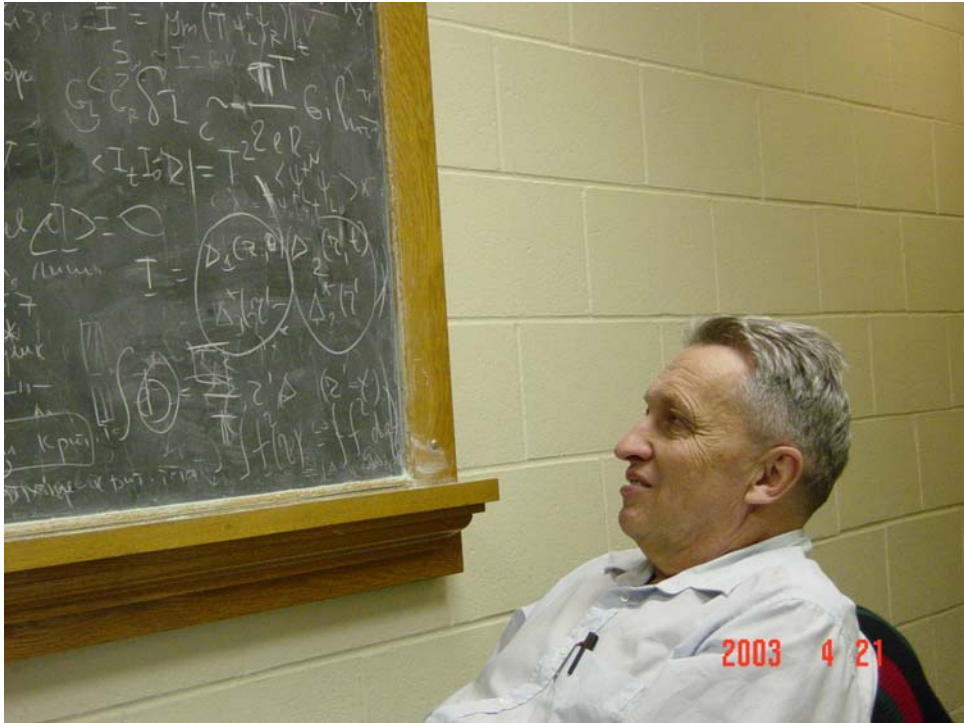
**A. I. LARKIN MEMORIAL CONFERENCE**

**June 24-28, 2007, Chernogolovka, Russia**

# **Fluctuation Phenomena in Tunnel Structures**

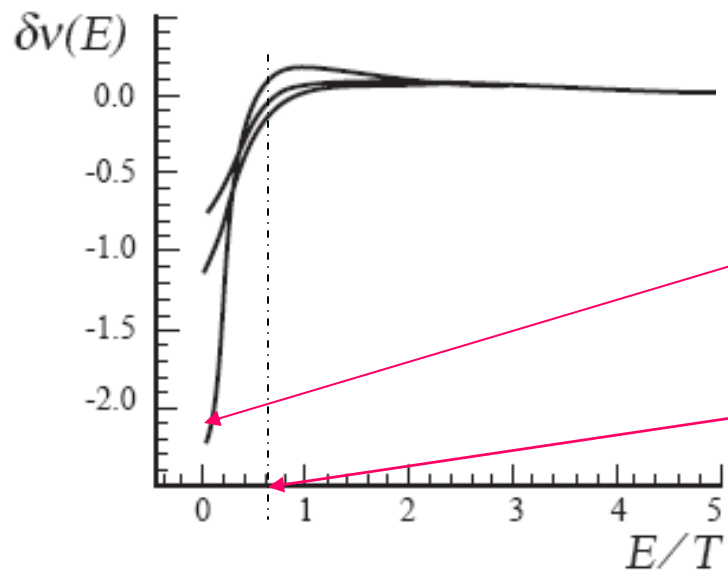
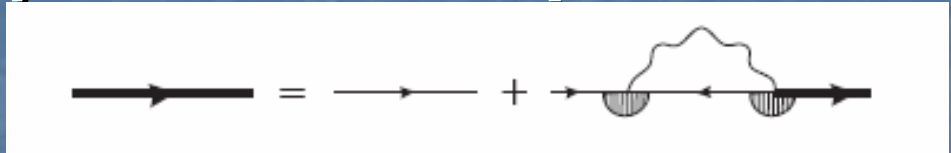
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# Fluctuation Renormalization of the density of states (Redi et al. 1970)



$$\frac{\delta\nu_{(D)}^{(d)}(0, \epsilon)}{\nu_{(D)}} \sim - \begin{cases} \sqrt{Gi_{(3,d)}} \epsilon^{-3/2}, & D = 3, \\ Gi_{(2,d)} \epsilon^{-2}, & D = 2. \end{cases}$$

$$t_{\xi}^{-1} = \mathcal{D}\xi^{-2}(T) \sim \tau_{\text{GL}}^{-1} \sim T - T_c$$

$$\int_0^{\infty} \delta\nu(E) dE = 0$$

# Tunneling Hamiltonian: Ambegaokar-Baratoff formula

$$I_T = e \left\langle \frac{d\hat{N}_L(t)}{dt} \right\rangle$$

$$\hat{\mathcal{H}}_T = \sum_{\mathbf{p}, \mathbf{k}} \left( T_{\mathbf{p}, \mathbf{k}} \hat{a}_{\mathbf{p}}^+ \hat{b}_{\mathbf{k}} + T_{\mathbf{k}, \mathbf{p}}^* \hat{a}_{\mathbf{p}} \hat{b}_{\mathbf{k}}^+ \right)$$

$$I_{qp}(V) = \frac{1}{eR_n \nu_L \nu_R} \int_{-\infty}^{\infty} \left( \tanh \frac{E + eV}{2T} - \tanh \frac{E}{2T} \right) \nu_R(E) \nu_L(E + eV) dE,$$

# Zero bias anomaly in disordered metal (Altshuler, Aronov, 1979)

$$G_{qp}(V) = \frac{I_{qp}(V)}{dV} = \frac{1}{2TR_n\nu_R} \int_{-\infty}^{\infty} \cosh^{-2} \left( \frac{E + eV}{2T} \right) \nu_R(E) dE.$$

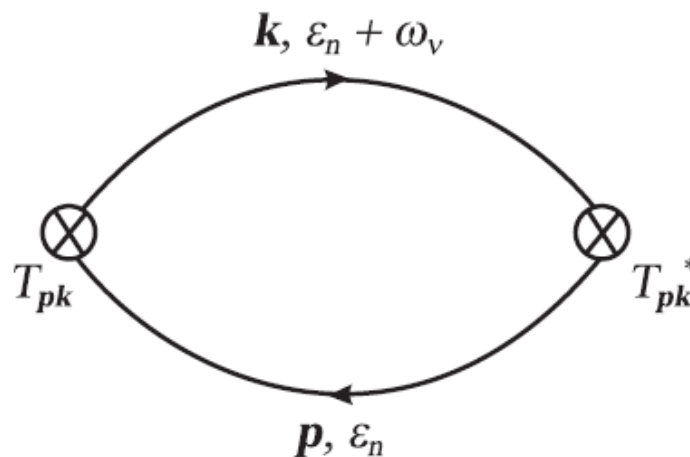
$$\tau^{-1} \gg T$$

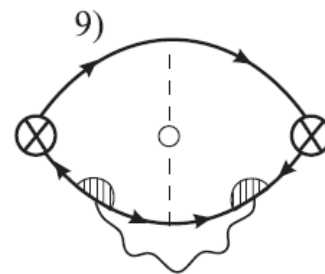
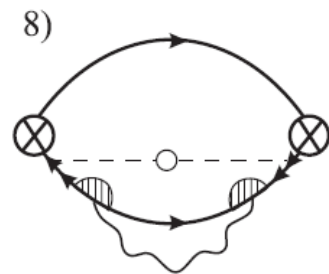
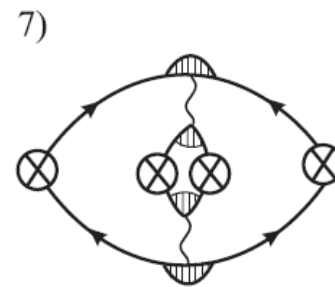
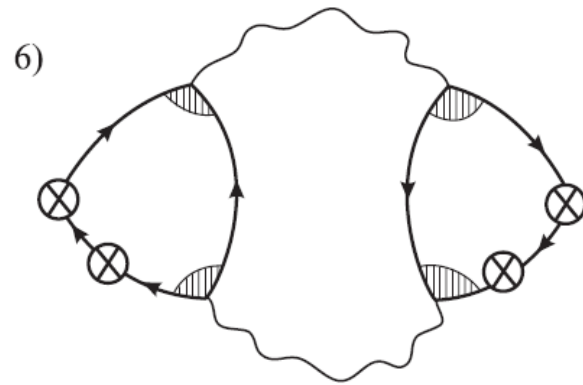
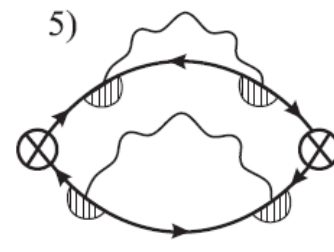
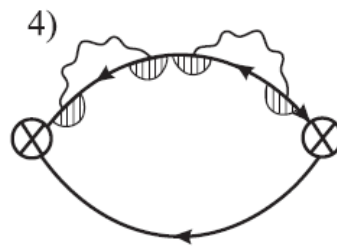
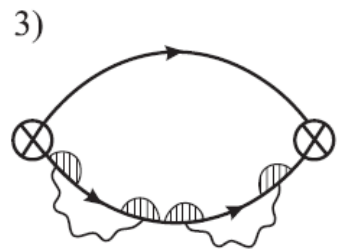
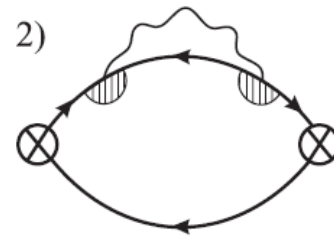
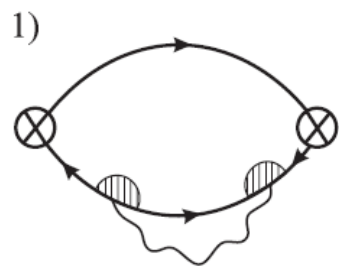
$$\frac{\delta G(V)}{G_n(0)} = \frac{\delta \nu(eV)}{\nu(0)}.$$

# Fluctuation Pseudogap in N(S)-I-N Junction (Varlamov, Dorin, 1983)

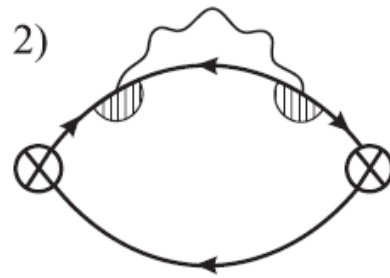
$$I_T(V) = -e \operatorname{Im} K^R (\omega_\nu \rightarrow -ieV) ,$$

$$K (\omega_\nu) = 4T \sum_{\epsilon_n} \sum_{\mathbf{p}, \mathbf{k}} |T_{\mathbf{p}, \mathbf{k}}|^2 G_R (\mathbf{p}, \epsilon_n + \omega_\nu) G_L (\mathbf{k}, \epsilon_n)$$

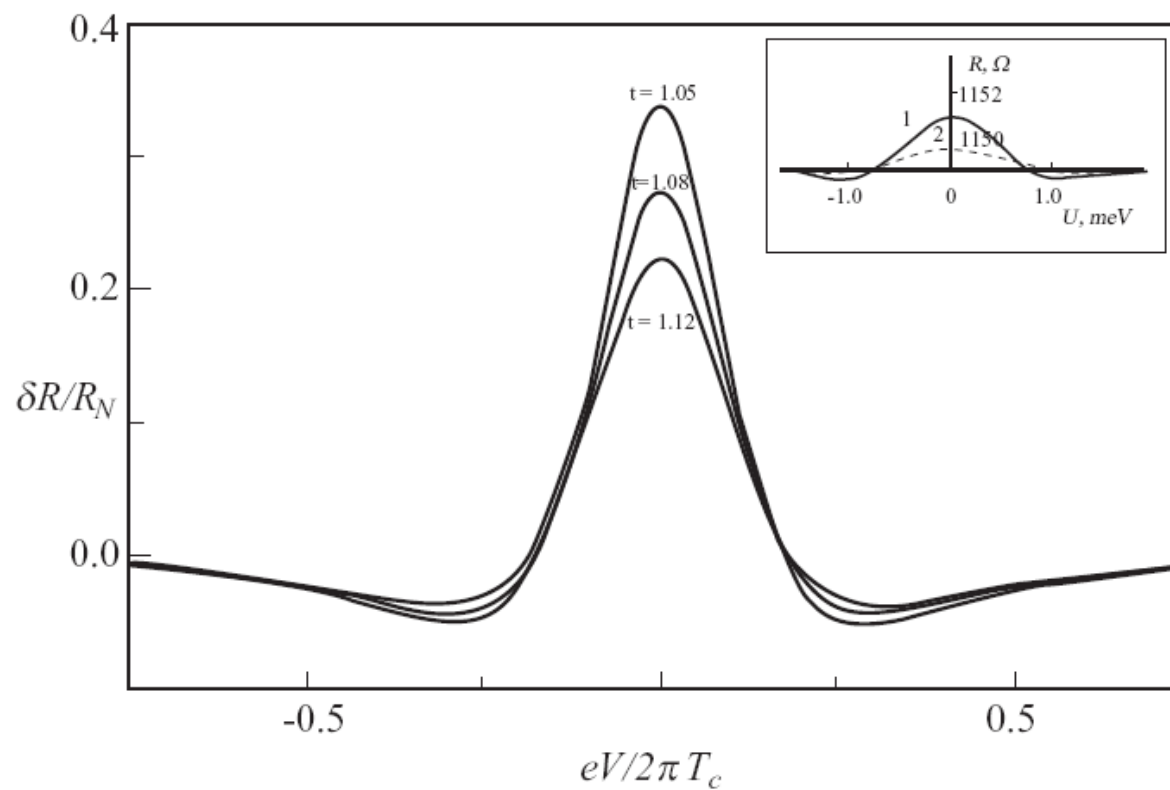




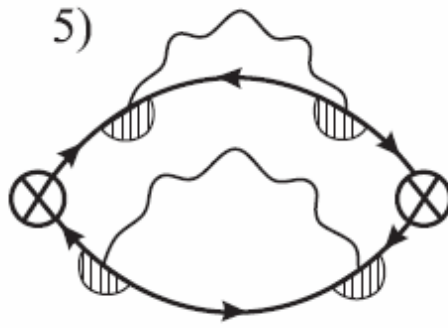




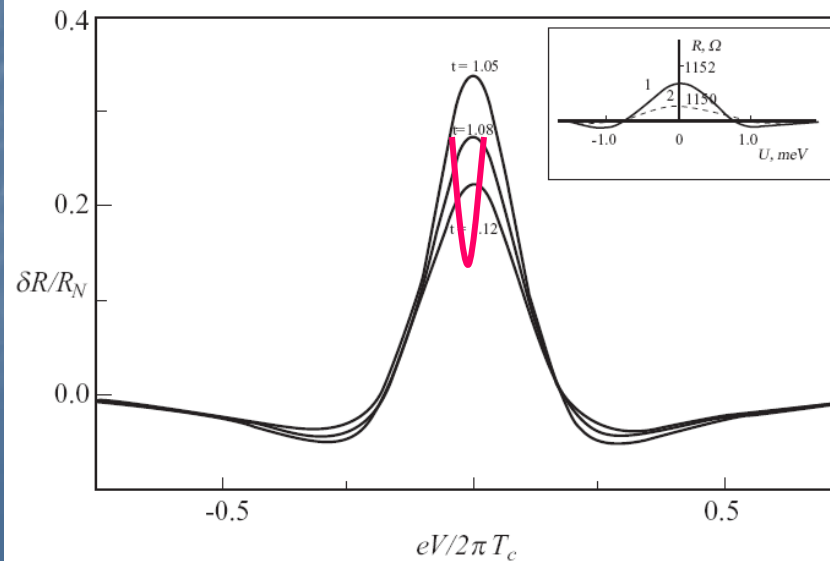
$$G_{qp}^{(1)}(V) = \frac{Gi(2d)}{7\zeta(3)R_n} \left( \ln \frac{1}{\epsilon} \right) \operatorname{Re} \psi'' \left( \frac{1}{2} - \frac{ieV}{2\pi T} \right)$$







$$\delta G_{\text{fl}}^{(2d)}(0, \epsilon) \sim \int_{-\infty}^{\infty} \frac{dE}{\cosh^2\left(\frac{E}{2T}\right)} \left[ \delta \nu_{(2)}^{(d)}(E, \epsilon) \right]^2 \sim \frac{G i_{(2d)}^2}{\epsilon^3}.$$



# Fluctuation Phenomena in N(S)-I-S Junction

Quasiparticle current

$$K(\omega_\nu) = T \sum_{\varepsilon_n} \sum_{\mathbf{p}_1, \mathbf{k}_1} \sum_{\mathbf{p}_2, \mathbf{k}_2} T_{\mathbf{k}_1 \mathbf{p}_1} G_n(\mathbf{p}_1, \mathbf{p}_2, \varepsilon_n + \omega_\nu) T_{\mathbf{p}_2 \mathbf{k}_2}^* G_S(\mathbf{k}_2, \mathbf{k}_1, \varepsilon_n)$$

$$I_{qp}(V) = \frac{2C_{NN}}{e} \Delta_2 \sum_{n=1}^{\infty} (-1)^{n+1} K_1\left(\frac{n\Delta_2}{T}\right) \sinh\left(\frac{neV}{T}\right)$$

At low temperatures is exponentially small

# Current related with Cooper pairs tunneling

## Boundary Hamiltonian

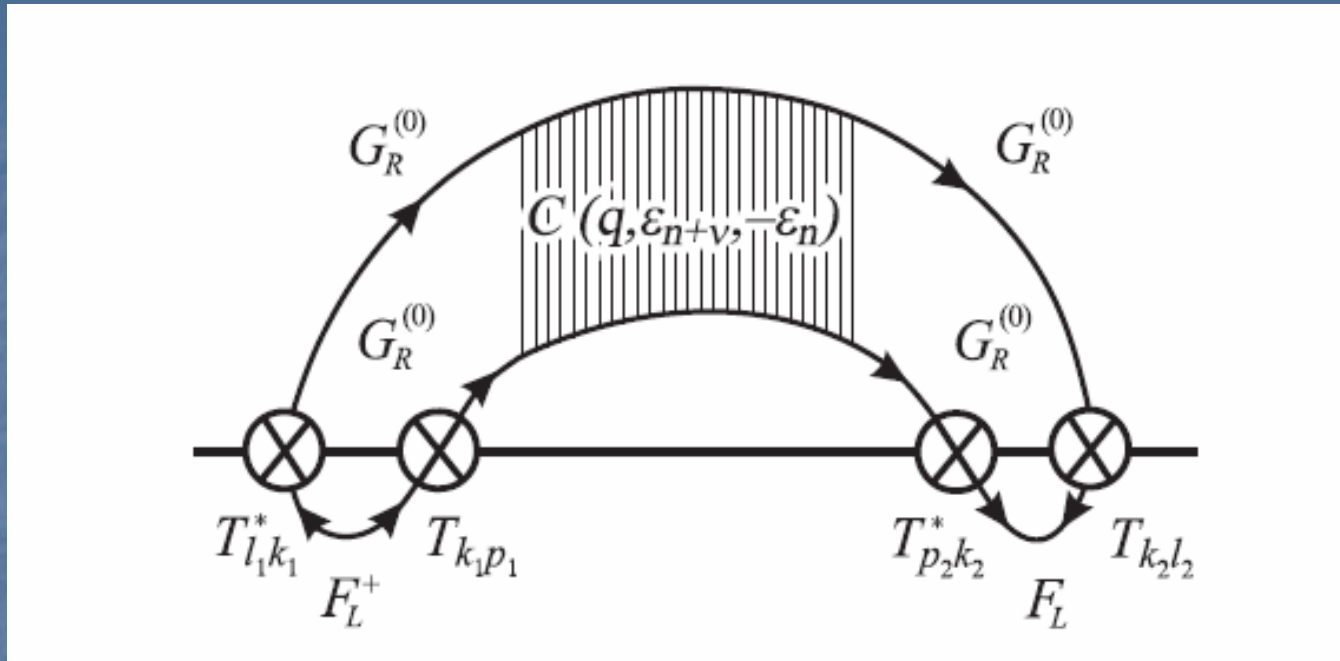
$$\hat{\mathcal{H}}_b = \frac{1}{8e^2 R_n} \int \exp(i\mathbf{q}\mathbf{r}) d^2\mathbf{r} \times \sum_{\mathbf{q}} \left( e^{-i\varphi(\mathbf{r})} \hat{a}_{\mathbf{p}+\mathbf{q}/2\uparrow} \hat{a}_{\mathbf{p}-\mathbf{q}/2\downarrow} + e^{i\varphi(\mathbf{r})} \hat{a}_{\mathbf{p}+\mathbf{q}/2\uparrow}^+ \hat{a}_{\mathbf{p}-\mathbf{q}/2\downarrow}^+ \right)$$

$$I_b = e \int_{-\infty}^0 \left\langle \left[ \hat{\mathcal{H}}_b(t), \left[ \sum_{\mathbf{p}, \sigma = \pm 1/2} \hat{a}_{\mathbf{p}\sigma}^+ \hat{a}_{\mathbf{p}\sigma}, \hat{\mathcal{H}}_b(0) \right] \right] dt \right\rangle$$

$$I_b(V, H) = -\frac{e}{(4\nu e^2 R_n)^2} \text{Im} \sum_{\mathbf{D}, \mathbf{D}'} \mathcal{L}^R(\mathbf{p}, \mathbf{p}', \mathbf{q}, 2eV).$$

$$\begin{aligned} L^R(\mathbf{q}, 2eV) &= \sum_{\mathbf{p}, \mathbf{p}'} \mathcal{L}^R(\mathbf{p}, \mathbf{p}', \mathbf{q}, 2eV) \propto \frac{\Pi^R}{1 - g\Pi^R} \\ &= \Pi^R + \Pi^R \frac{1}{1/g - \Pi^R} \Pi^R. \end{aligned}$$

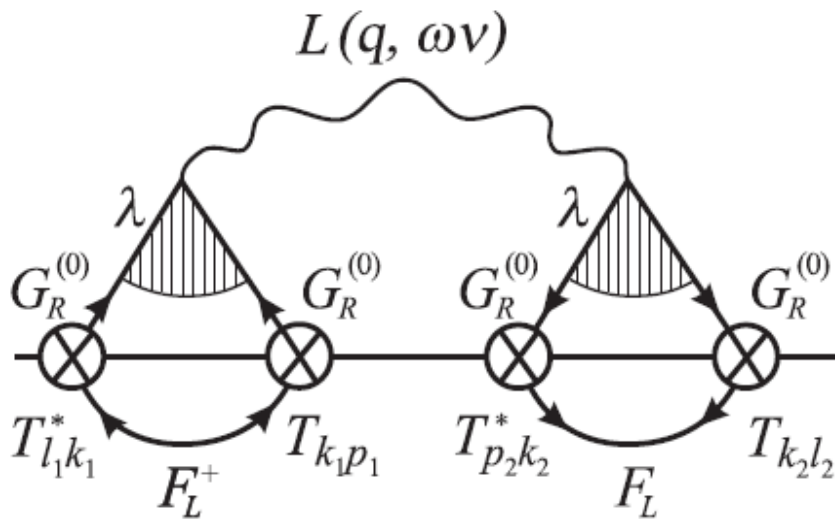
# Andreev's conductance (Volkov et al, 1993)



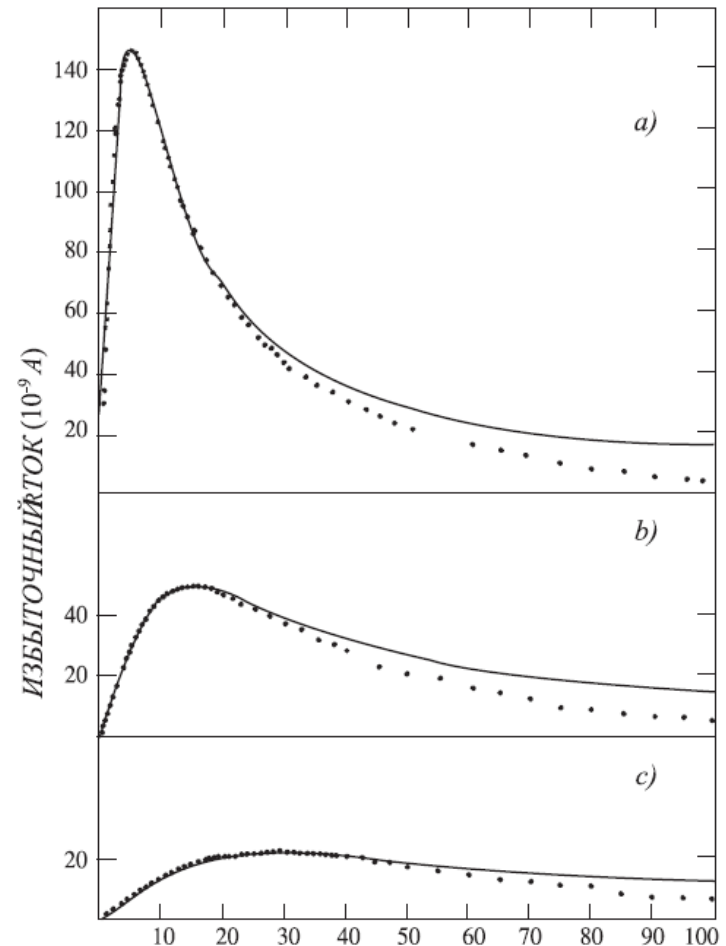
$$\text{Im } \Pi (2eV) = -\nu \text{ Im } \psi \left( \frac{1}{2} + \frac{-2ieV + \mathcal{D}q^2}{4\pi T} \right)$$



# Superconducting "pair field" susceptibility (Scalapino, 1970)

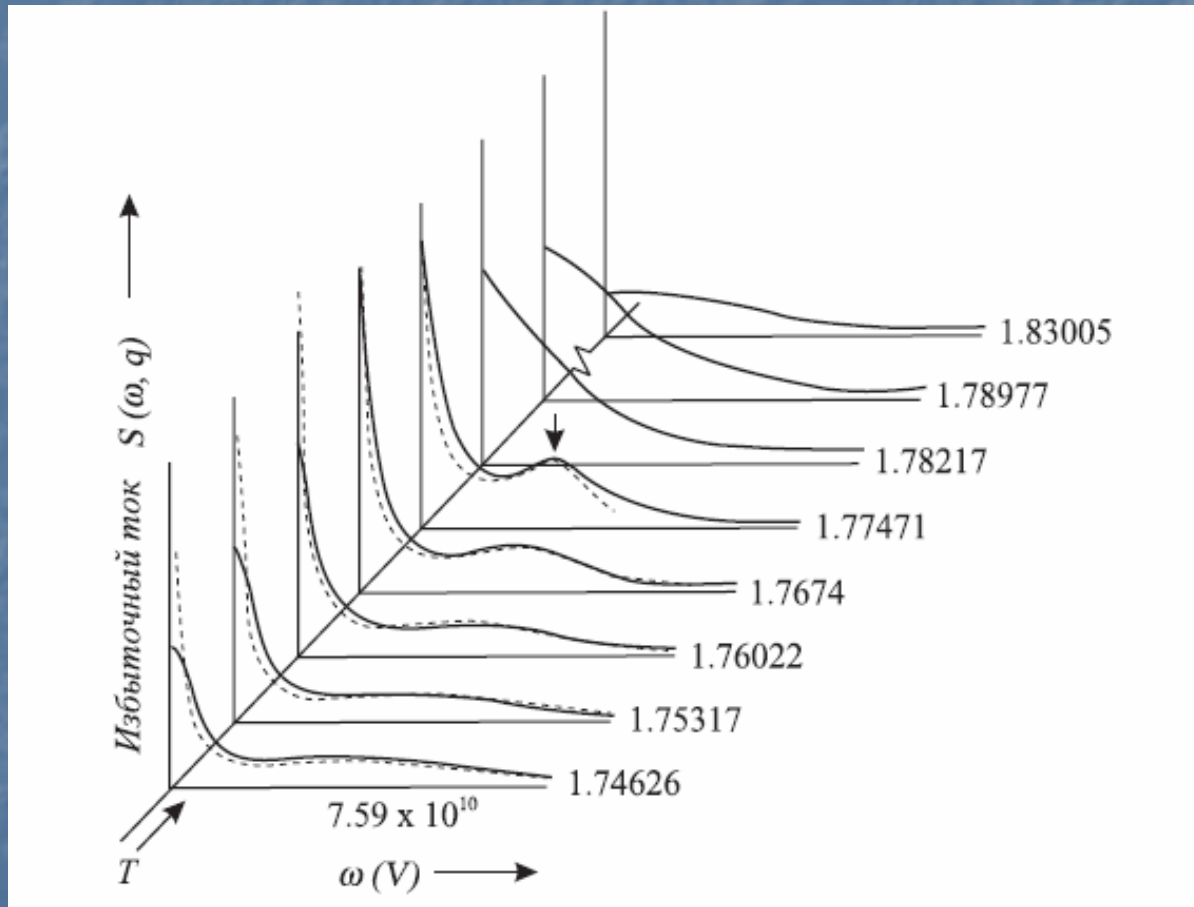


$$\omega = 2eV \text{ и } q = 2eH\lambda_L.$$



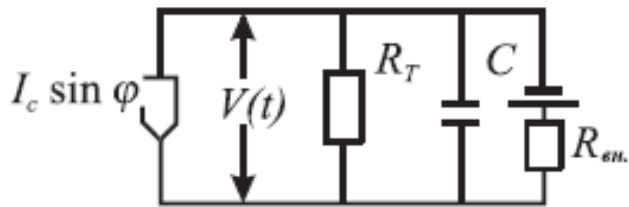
$$I_{\parallel}(q, \omega) = \frac{S}{16e^3 R_n^2 d \epsilon} \ln^2 \frac{\Delta_2}{T} \frac{\omega \tau_{GL}}{(1 + \xi^2 q^2)^2 + \omega^2 \tau_{GL}^2},$$

# Collective modes in SC (Goldman et al, 1973)



# Fluctuations in Josephson structures

Fluctuation smearing of the emission line (Larkin, Ovchinnikov, 1967)



$$Z^{-1}(\omega) = -i\omega C + R_T^{-1} + R_{ext}^{-1}$$

$$Q = \Gamma R^* C.$$

$$K(\omega) = \frac{2}{\pi} \int_{-\infty}^{\infty} e^{-i\omega\tau} \overline{I(t)I(t+\tau)} d\tau.$$

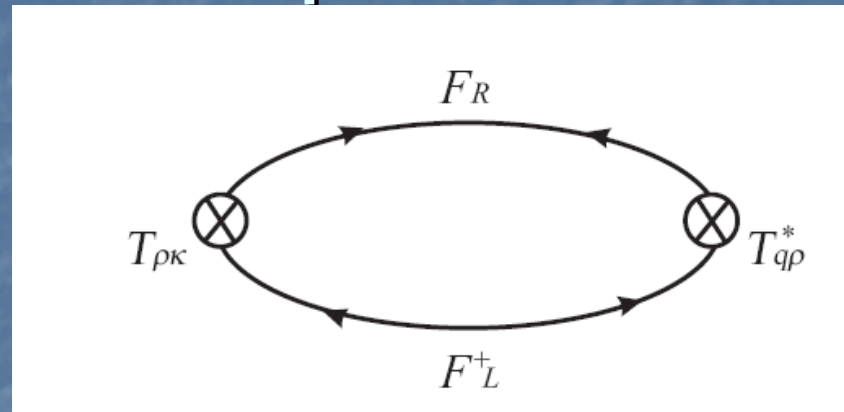
$$K(\omega) \sim \frac{1}{\pi} \frac{\Gamma}{(\omega - \omega_J)^2 + \Gamma^2}.$$

$$Q \ll 1,$$

$$K(\omega) \sim \frac{1}{\sqrt{\pi}\tilde{\Gamma}} \exp \left[ - \left( \frac{\omega - \omega_J}{\tilde{\Gamma}} \right)^2 \right], \quad \tilde{\Gamma} = 2e \sqrt{\frac{2\Gamma^*}{C}}$$

$$Q \gg 1$$

# Thermal fluctuations of the order parameter



$$I_c = 4e \sum_{\mathbf{p}, \mathbf{q}} |T_{\mathbf{p}\mathbf{q}}|^2 T \sum_{\varepsilon_n} F_L^+(p, \varepsilon_n) F_R(q, -\varepsilon_n),$$

$$I_c = \frac{\pi}{eR_n} T \sum_{\varepsilon_n} \frac{\Delta_L}{\sqrt{\Delta_L^2 + \varepsilon_n^2}} \frac{\Delta_R}{\sqrt{\Delta_R^2 + \varepsilon_n^2}},$$

$$I_c = \frac{\pi\Delta(T)}{2eR_n} \tanh \frac{\Delta(T)}{2T}.$$



# Asymmetric junction (Larkin, Varlamov, 2004)



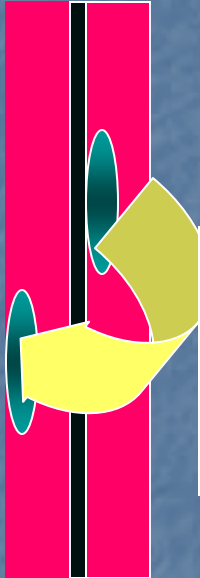
$$I_c(t) = \frac{\pi}{eR} T \sum_n \int d^2\mathbf{r} \frac{\Delta_1^{(fl)}(\mathbf{r}, t)}{|\epsilon_n|} \frac{\Delta_2}{\sqrt{\Delta_2^2 + \epsilon_n^2}}$$

$$K(\omega) = \frac{2}{\pi} F_c^2 \left( \frac{\Delta_2(T_{c1})}{2\pi T_{c1}} \right) \int d^2\mathbf{r} d^2\mathbf{r}' \times \\ \times \int_{-\infty}^{\infty} e^{-i(\omega - \omega_J)\tau} \langle \Delta^{(fl)}(\mathbf{r}, t) \Delta^{*(fl)}(\mathbf{r}', t + \tau) \rangle d\tau =$$

$$\text{Re } K(\omega) = \frac{64 T_{c1}^2 S}{\pi^2 \nu e^2 R^2} \left( \ln^2 \frac{\Delta_2(T_{c1})}{\pi T_{c1}} \right) \frac{\Gamma(\epsilon)}{\Gamma^2(\epsilon) + (\omega - \omega_J)^2}$$

Symmetric junction (small junction, Kulik, 1970;  
large junction, Larkin, Varlamov, 2004)

$$I_c(t) = \frac{\pi}{eR} T \sum_n \int d^2\mathbf{r} \frac{\Delta_1^{(fl)}(\mathbf{r}, t)}{\sqrt{\Delta_1^{(fl)2} + \varepsilon_n^2}} \frac{\Delta_2^{(fl)}(\mathbf{r}, t)}{\sqrt{\Delta_2^{(fl)2} + \varepsilon_n^2}}$$

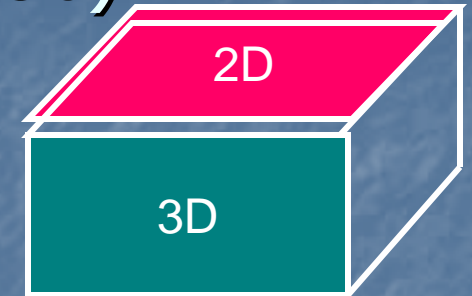


$$K(\omega) = \frac{\pi S}{2e^2 R^2 T^2} \left[ \frac{1}{4m\nu\xi^2} \right]^2 \int_{-\infty}^{\infty} e^{-i(\omega - \omega_J)\tau} \int d^2\mathbf{q} \langle \Psi \Psi^* \rangle_{t, q \rightarrow 0}^2 d\tau$$

$$\text{Re } K(\omega) = \frac{16\pi^3 TS}{7\zeta(3) e^2 R^2 \nu} Gi_{(2)} \ln \frac{32T\Gamma(\epsilon) / \pi}{4\Gamma^2(\epsilon) + (\omega - \omega_J)^2}$$

# Fluctuation renormalization of the Josephson current (Varlamov, Dorin, 1986)

$$I_c = \frac{\pi \Delta^2 (T)}{4eT R_n}$$



Mean field in the vicinity of  $T_c$  gives:

$$\Delta^2 (T) \rightarrow \Delta_0^2 (T) = \frac{8\pi^2 T_{c0}^2}{7\zeta(3)} |\epsilon_0|,$$

$$I_c^{(0)}(|\epsilon_0|) = \frac{2\pi^3 T_{c0}}{7\zeta(3) e R_n} |\epsilon_0|$$

$$\text{where } |\epsilon_0| = (T_{c0} - T) / T_{c0}$$

GL fluctuation theory gives:

$$\Delta^2(T) \rightarrow \langle \Delta(T) \rangle^2.$$

$$\langle \Delta(T) \rangle_{(\text{fl})}^2 = \Delta_0^2 - \frac{1}{4mC_{(D)}} [3 \langle \psi_r^2 \rangle + \langle \psi_i^2 \rangle]$$

$$I_c(T) = \frac{\pi \langle \Delta(T) \rangle_{(\text{fl})}^2}{4eTR_n} = I_c^{(0)}(|\epsilon_0|) + \delta I_c^{(\text{fl})}(|\epsilon_0|).$$

$$\delta I_{c(2)}(|\epsilon|) = \frac{4\pi^3 T_c}{7\zeta(3) eR_n} G_{i(2)} \left[ \ln \frac{|\epsilon|}{G_{i(2)}} + \ln \left( \frac{\xi(T)}{L_J} \right) \right].$$



# Exponential tail in Josephson junction close to $T_c$ (Larkin, Varlamov, 2004)

$$\mathcal{F}[\varphi] = \frac{n_s}{4m} \int d^2\mathbf{r} [\nabla\varphi(\mathbf{r})]^2 = \frac{n_s}{4m} \sum_{\mathbf{k}} k^2 \varphi_{\mathbf{k}}^2.$$

+

$$\delta E_J^{(\text{fl})}(\epsilon) = \frac{E_J}{2S} \int d^2\mathbf{r} \left[ \varphi^{(\text{fl})}(\mathbf{r}) \right]^2.$$

$$F\left(\varphi^{(\text{fl})}\right) = \frac{n_s}{2m} \sum_{\mathbf{k}} \left( k^2 + \frac{1}{L_J^2} \right) \varphi_{\mathbf{k}}^2,$$

where the Josephson length  $L_J$  is determined by relation

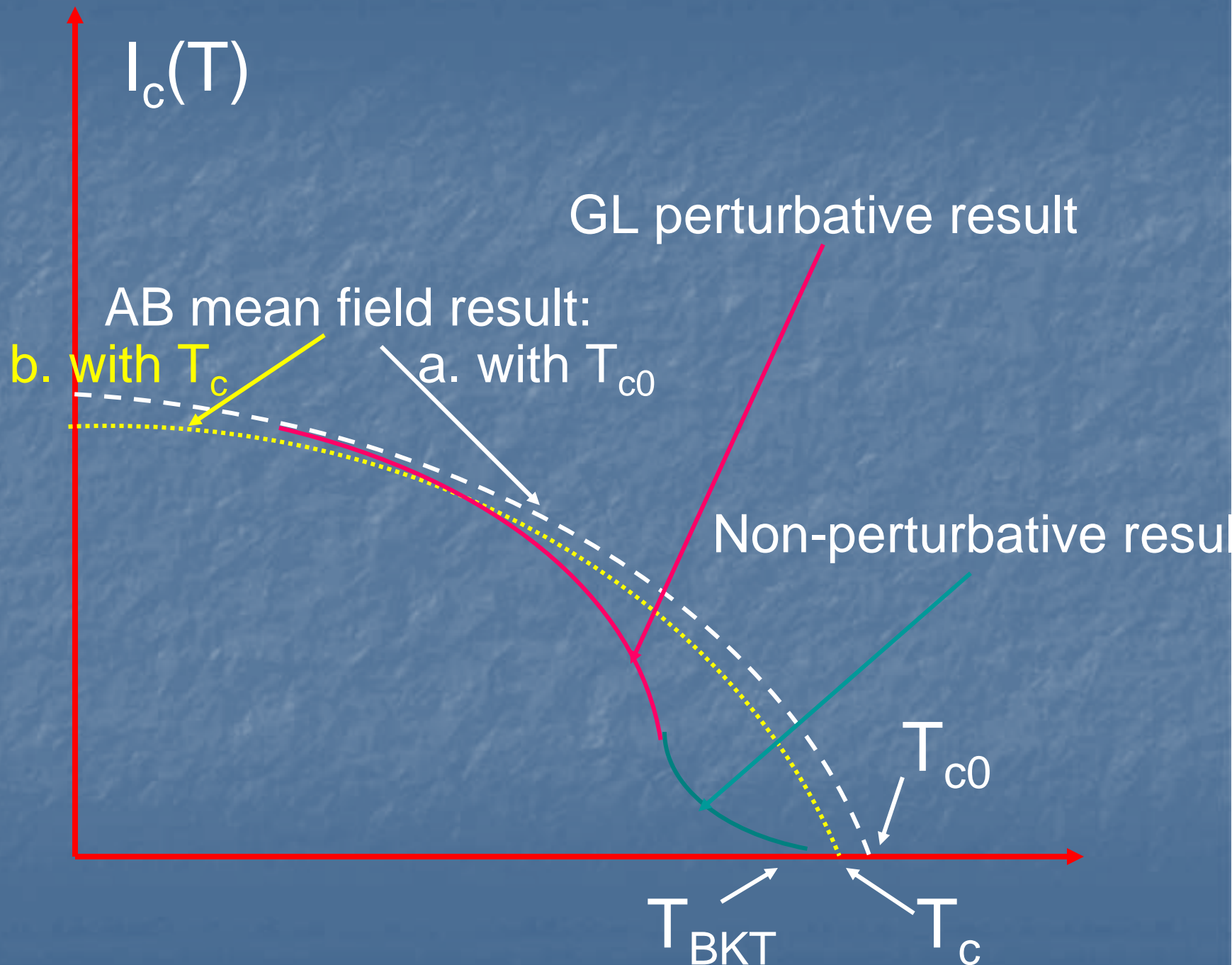
$$\frac{n_s}{2mL_J^2} = \frac{E_J}{2S}.$$

$$I_c(\varphi) = \frac{E_J}{2S} \left\langle \sin \left( \varphi + \varphi^{(\text{fl})} \right) \right\rangle$$

$$\delta E_J^{(\text{fl})}(\epsilon) = E_{J0} \exp \left( -\frac{2Gi_{(2d)}}{|\epsilon|} \ln \frac{L_J}{\xi(T)} \right)$$

$$I_c(\epsilon) = I_c^{(AB)}(\epsilon) \exp \left( -\frac{\epsilon^*}{|\epsilon|} \right)$$

$$\epsilon^* = Gi_{(2d)} \ln \left[ Gi_{(2d)} \left( \frac{L_J}{\xi} \right)^2 \right]$$



# Fluctuation spectroscopy of granularity in superconducting structures

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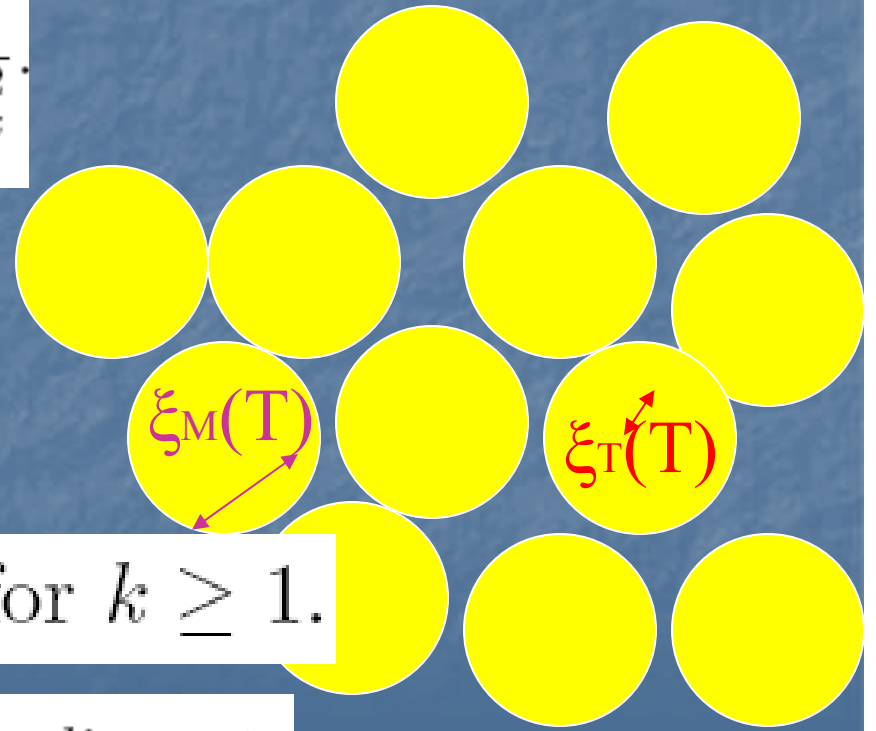
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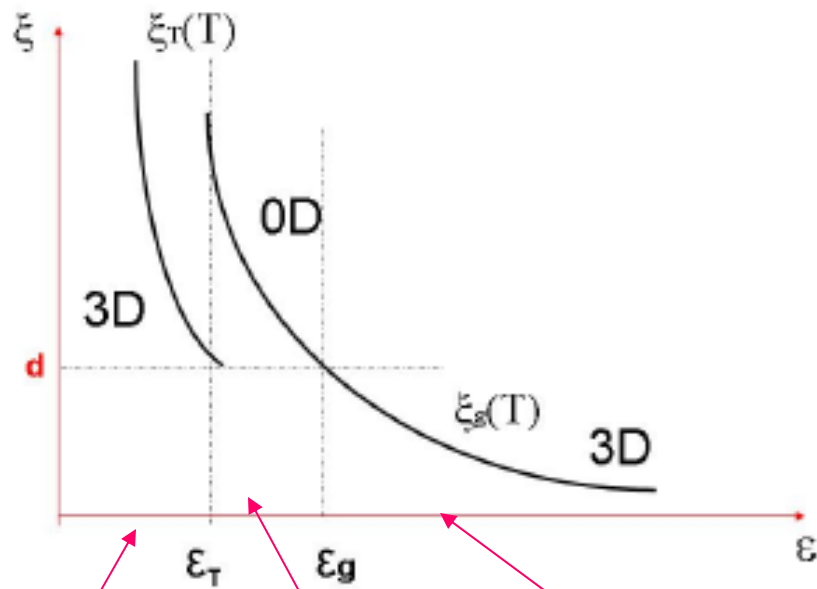
$$\mathcal{L}(q_k, \omega_m) = -\frac{1}{\nu} \frac{1}{\epsilon - i\pi\omega_m/8T_c + \xi_g^2 q_k^2}$$

$$q_k d = 2 \tan(q_k d/2)$$

$$q_0 = 0 \text{ and } q_k d \approx 2\pi k + \pi \text{ for } k \geq 1.$$

$\Gamma = g_T \delta$  is the intergrain tunneling rate





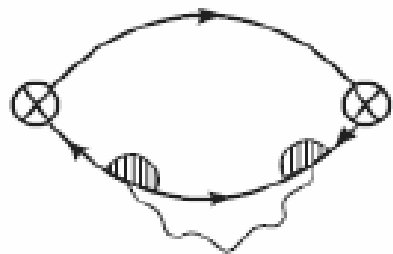
$$\xi_g(\epsilon) \equiv \frac{\xi_g}{\sqrt{\epsilon}}, \quad \xi_T(\epsilon) \equiv \frac{\xi_T}{\sqrt{\epsilon}}.$$

$$\epsilon \lesssim \epsilon_T,$$

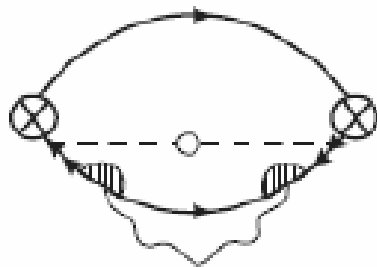
$$(\xi_g/d)^2 \equiv \epsilon_g \lesssim \epsilon \lesssim 1,$$

$$(\Gamma/T_c) \equiv \epsilon_T \lesssim \epsilon \lesssim \epsilon_g$$

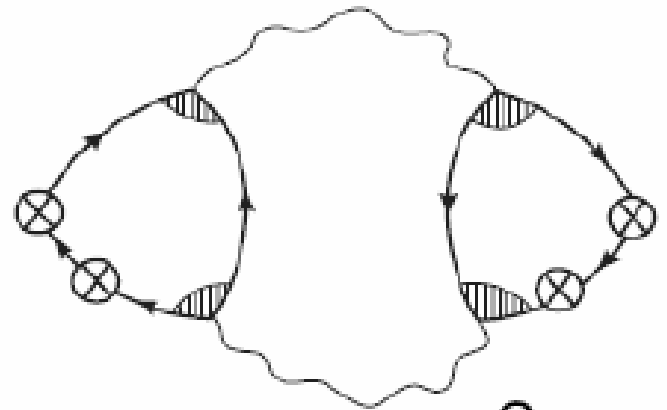




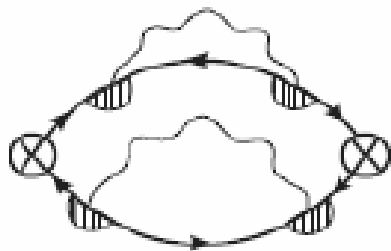
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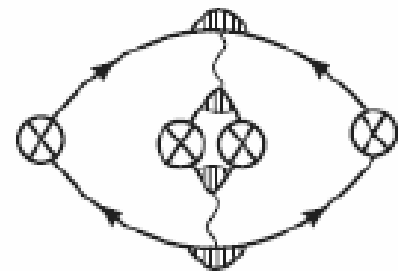
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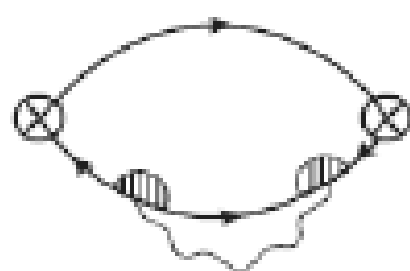
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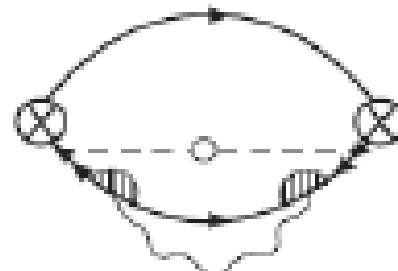
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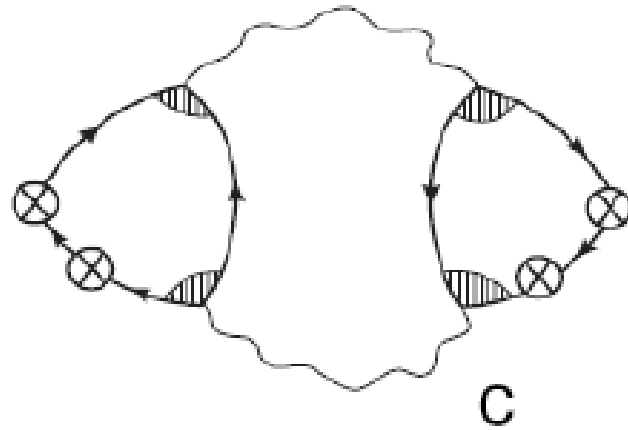


b

$$\sigma_{\text{DoS}}(\epsilon) \sim -\frac{e^2}{d} \left( \frac{\Gamma}{T_c} \right) \sum_{\mathbf{q}} \frac{1}{\epsilon + \xi_g^2 q_k^2}.$$

$$\sigma_{\text{DoS}}(\epsilon) \sim -\frac{e^2}{d} \times \begin{cases} \frac{\epsilon_T}{\sqrt{\epsilon_g \epsilon}}, & \epsilon_g \lesssim \epsilon \lesssim 1 \\ \frac{\epsilon_T}{\epsilon}, & \epsilon_T \lesssim \epsilon \lesssim \epsilon_g \end{cases}$$

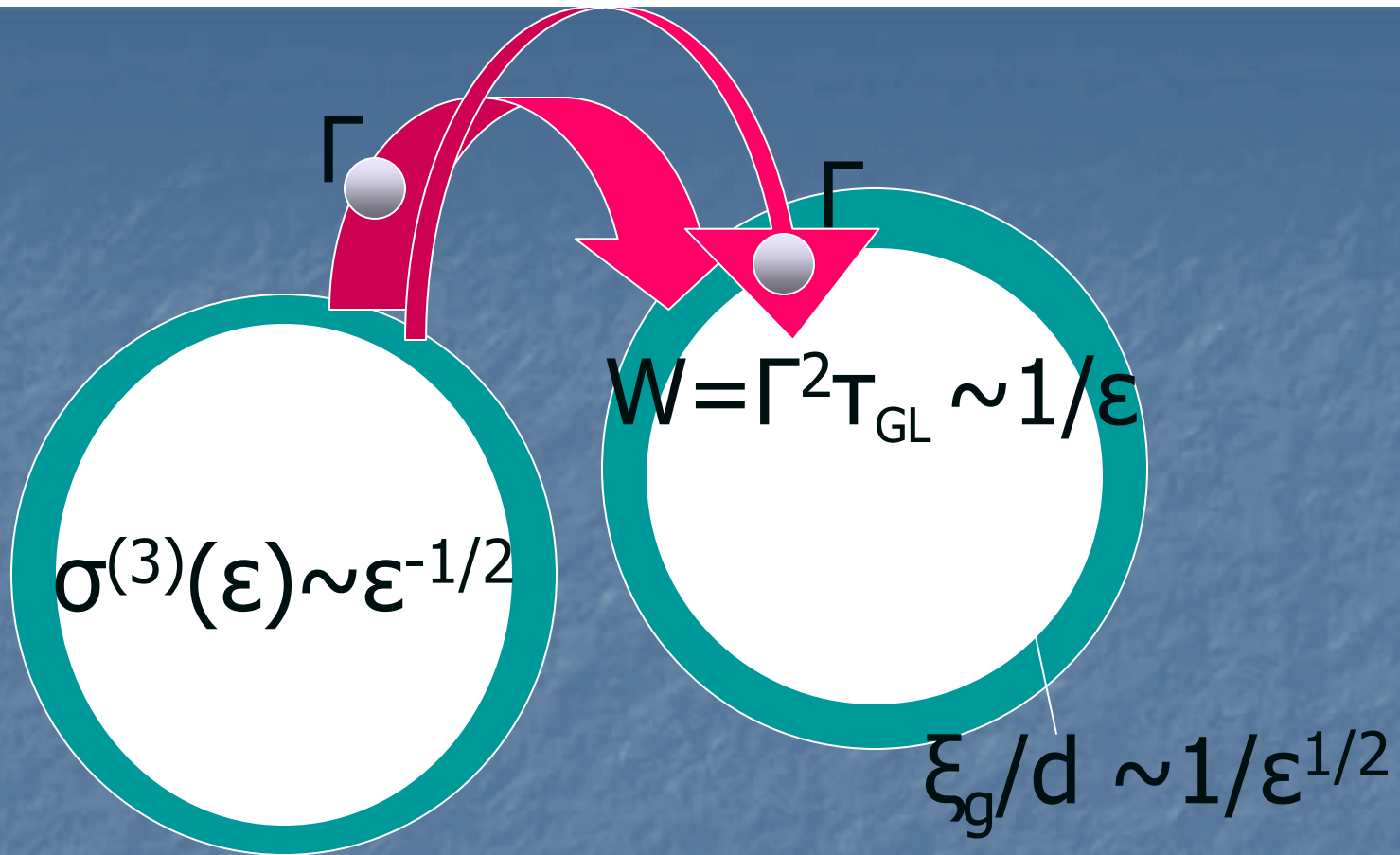
$$\epsilon_T \equiv \Gamma/T_c \ll 1$$



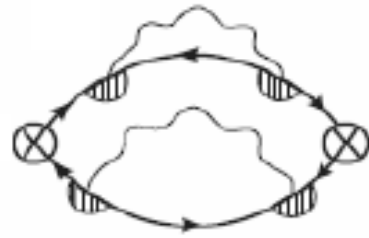
$$\sigma_{\text{AL}}(\epsilon) \sim \frac{\pi e^2}{d} \left( \frac{\Gamma}{T_c} \right)^2 \sum_{k,l} \frac{1}{(2\epsilon + \xi_g^2 q_k^2 + \xi_g^2 q_l^2) (\epsilon + \xi_g^2 q_k^2) (\epsilon + \xi_g^2 q_l^2)}$$

$$\sigma_{\text{AL}}(\epsilon) \sim \frac{e^2}{d} \left( \frac{\Gamma}{T_c} \right)^2 \int_{-\infty}^{\infty} d\zeta \left( \sum_{\mathbf{q}_k} \frac{1}{(\epsilon + \xi_g^2 q_k^2)^2 + \zeta^2} \right)^2$$

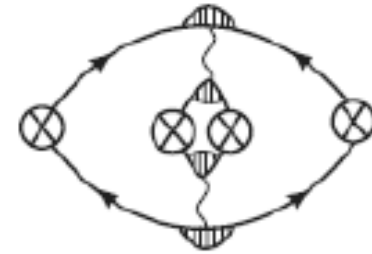
$$\sim \frac{e^2}{d} \times \begin{cases} \frac{\epsilon_{\text{T}}^2}{\epsilon_{\text{g}} \epsilon^2}, & \epsilon_{\text{g}} \lesssim \epsilon \lesssim 1 \\ \frac{\epsilon_{\text{T}}^2}{\epsilon^3}, & \epsilon_{\text{T}} \lesssim \epsilon \lesssim \epsilon_{\text{g}}. \end{cases}$$



$$\sigma^{(3t)}(\epsilon) = \sigma^{(3)}(\epsilon) \Gamma^2 \tau_{GL}(\epsilon) \xi_g(\epsilon)/d \sim 1/\epsilon^2$$



d



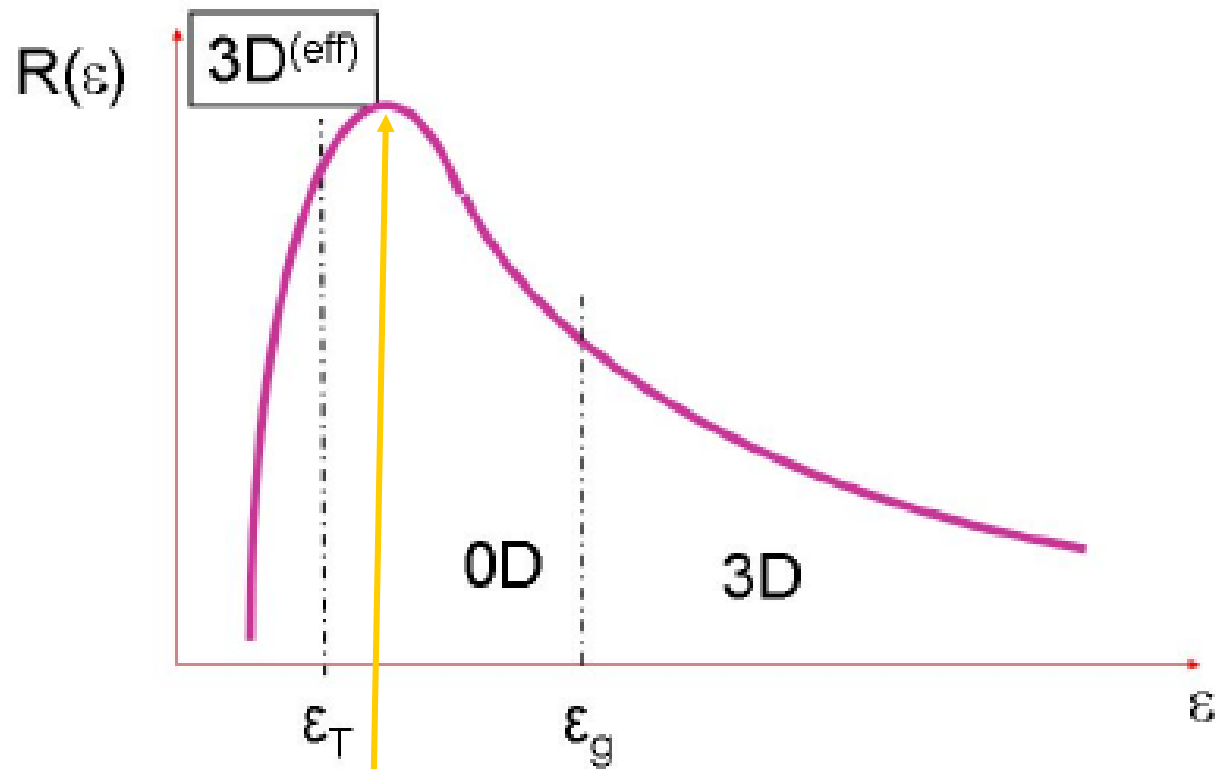
e

$$\sigma_{\text{MT}} \sim \frac{e^2}{d} \left( \frac{\delta}{T} \right) \left( \frac{\Gamma}{T_c} \right) \sum_{\mathbf{q}_k} \sum_{\mathbf{q}_l} \frac{1}{(\epsilon + \xi_g^2 q_k^2)} \times \frac{1}{(\epsilon + \xi_g^2 q_l^2)} \frac{1}{[2\gamma_\varphi + \xi_g^2 q_k^2 + \xi_g^2 q_l^2]^3},$$

$$\sigma_{\text{MT}}(\epsilon) \sim \frac{e^2}{d} \left( \frac{\delta}{T} \right) \frac{\epsilon_T}{\gamma_\varphi^3} \frac{1}{\epsilon^2} \times \begin{cases} \frac{\gamma_\varphi}{\epsilon_g}, & \epsilon_g \lesssim \gamma_\varphi \lesssim \epsilon \lesssim 1 \\ \frac{\epsilon}{\epsilon_g}, & \epsilon_g \lesssim \epsilon \lesssim \gamma_\varphi \lesssim 1 \\ 1, & \epsilon_T \lesssim \epsilon \lesssim \min(\epsilon_g, \gamma_\varphi) \end{cases}$$



$$\Gamma/T \gg (\delta/T)^{1/3}$$



$$\epsilon_{max} = (\Gamma/T)^{1/3}$$