Landau Institute for Theoretical Physics

A. I. LARKIN MEMORIAL CONFERENCE

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Fluctuation Phenomena in Tunnel Structures

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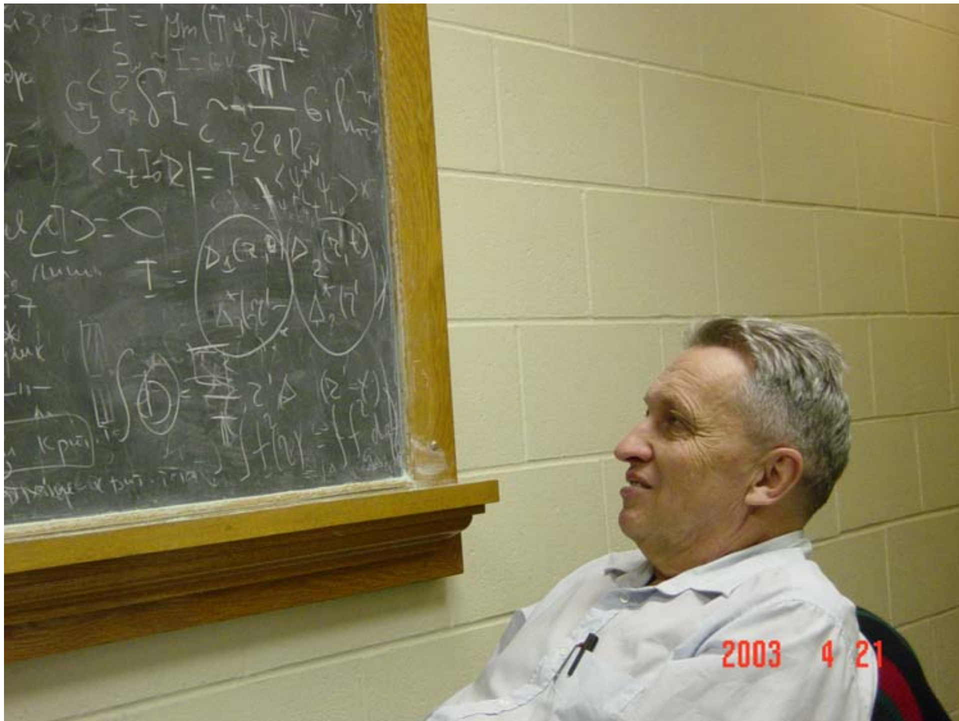
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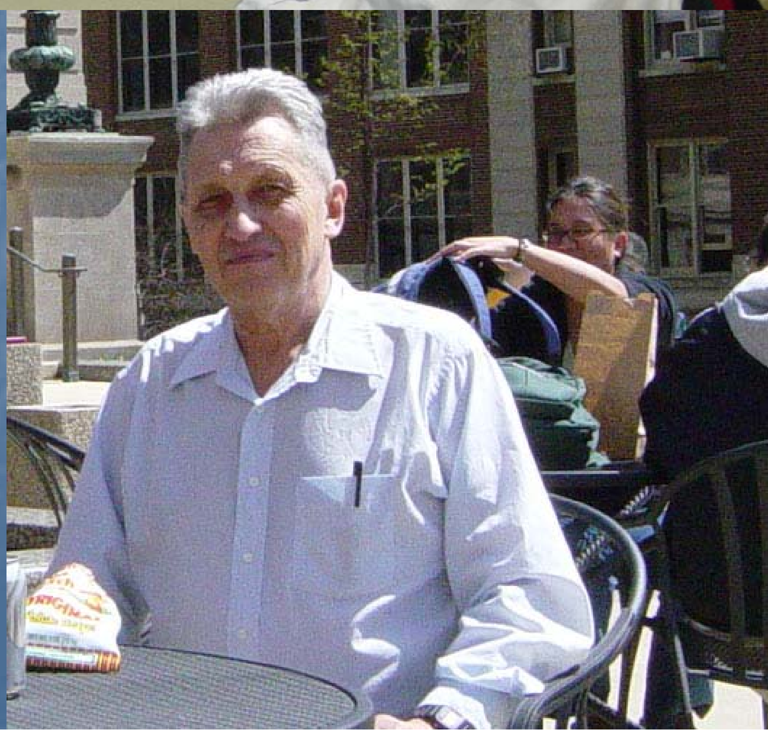
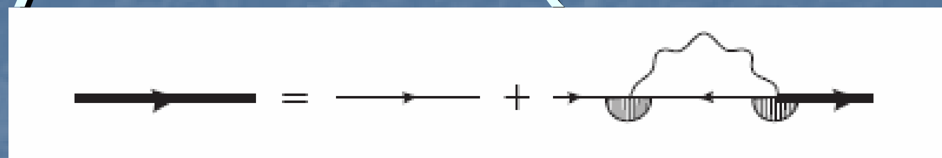
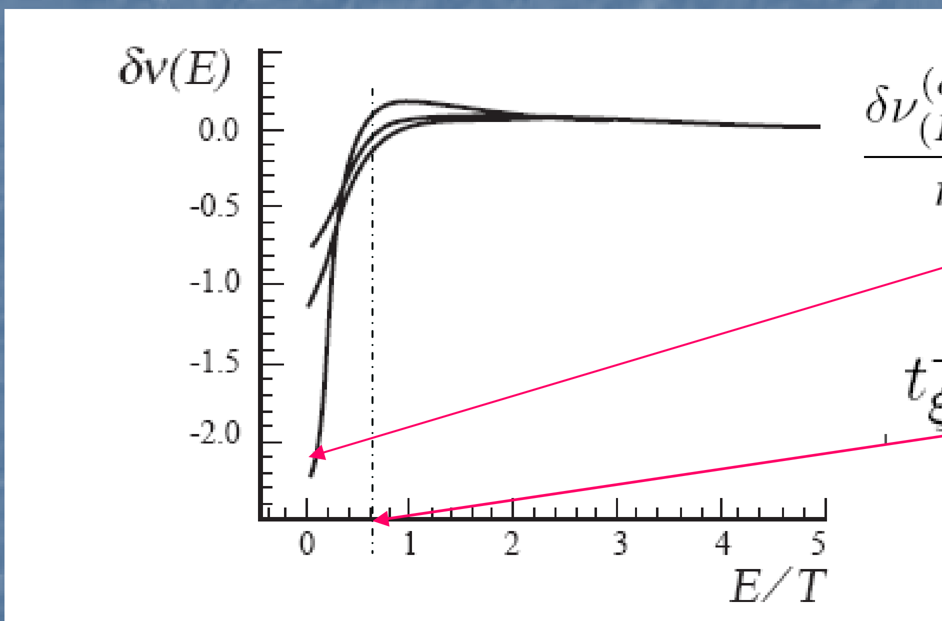


Fluctuation Renormalization of the density of states (Redi et al. 1970)





$$\frac{\delta\nu^{(d)}_{(D)}(0,\epsilon)}{\nu_{(D)}} \sim -\begin{cases} \sqrt{Gi_{(3,d)}}\,\epsilon^{-3/2}, & D=3, \\ Gi_{(2,d)}\,\epsilon^{-2}, & D=2. \end{cases}$$

$$t_{\xi}^{-1} = \mathcal{D}\xi^{-2}(T) \sim \tau_{\mathrm{GL}}^{-1} \sim T - T_{\mathrm{c}}$$

$$\int_0^{\infty} \delta\nu(E)\, dE = 0$$

Tunneling Hamiltonian: Ambegaokar-Baratoff formula

$$I_T = e \left\langle \frac{d\widehat{N}_L(t)}{dt} \right\rangle$$

$$\widehat{\mathcal{H}}_T = \sum_{\mathbf{p},\mathbf{k}} \left(T_{\mathbf{p},\mathbf{k}} \widehat{a}^{+}_{\mathbf{p}} \widehat{b}_{\mathbf{k}} + T^{*}_{\mathbf{k},\mathbf{p}} \widehat{a}_{\mathbf{p}} \widehat{b}^{+}_{\mathbf{k}} \right)$$

$$I_{qp}(V) = \frac{1}{eR_n \nu_L \nu_R} \int_{-\infty}^{\infty} \left(\tanh \frac{E+eV}{2T} - \tanh \frac{E}{2T} \right) \nu_R(E) \nu_L(E+eV) dE,$$

Zero bias anomaly in disordered metal (Altshuler, Aronov, 1979)

$$G_{qp}(V) = \frac{I_{qp}\ (V)}{dV} = \frac{1}{2TR_n\nu_R} \int\limits_{-\infty}^{\infty} \cosh^{-2}\left(\frac{E+eV}{2T}\right)\ \nu_R(E)dE.$$

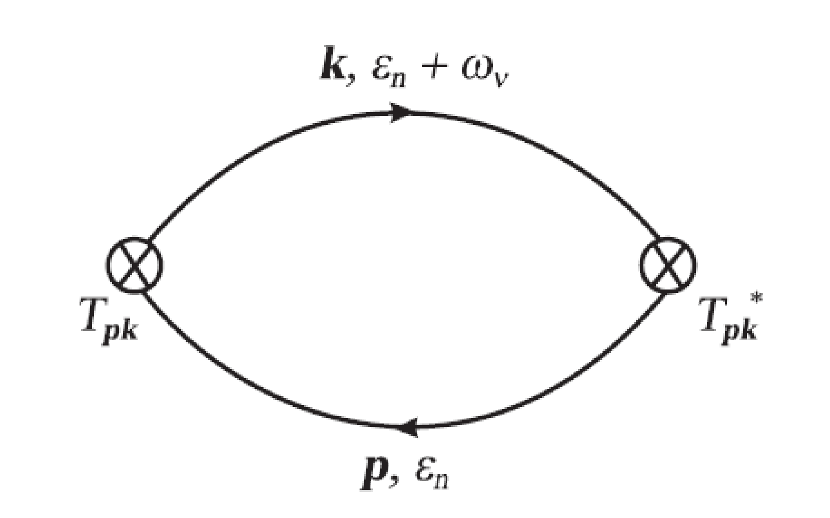
$$\tau^{-1} \gg T$$

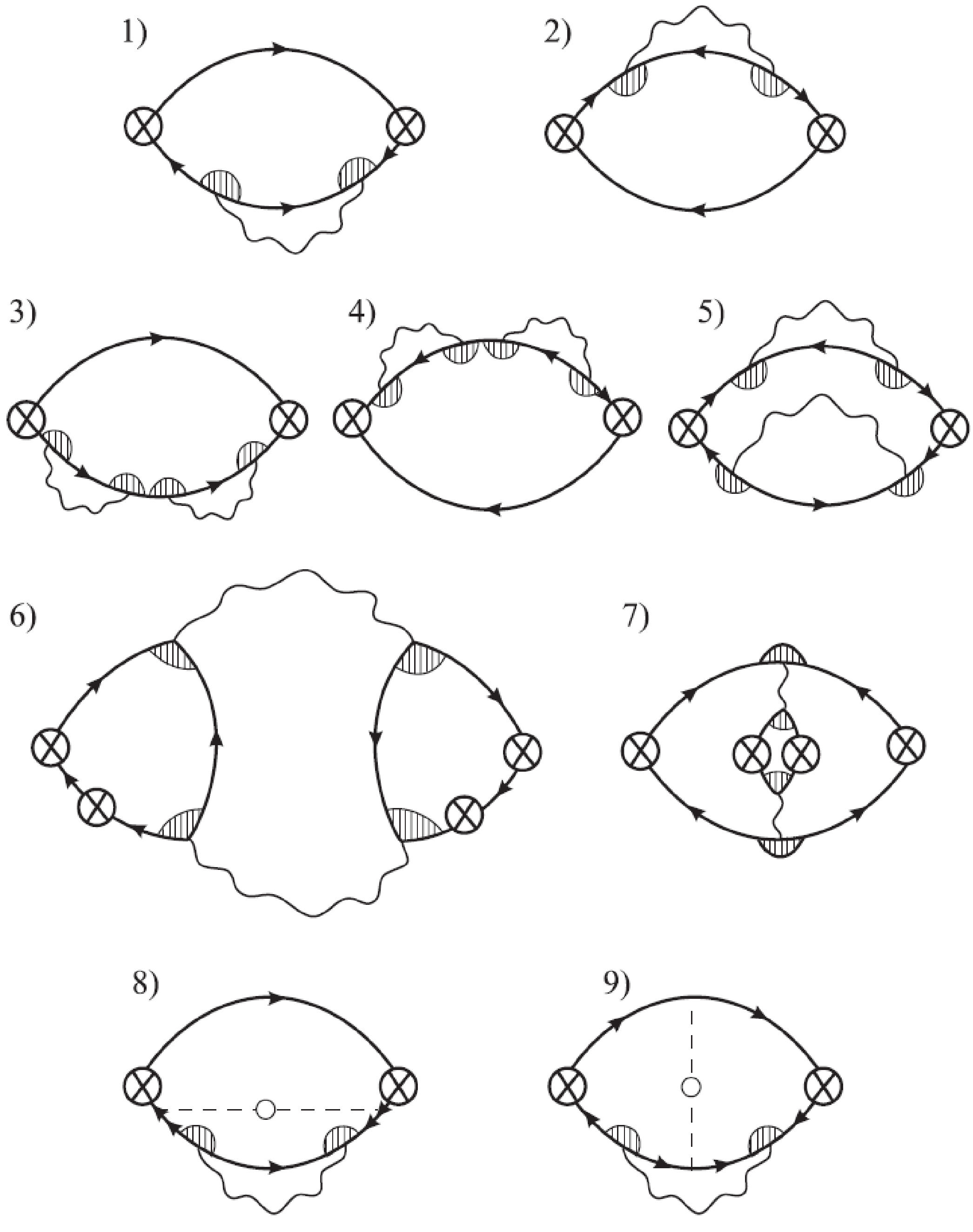
$$\frac{\delta G(V)}{G_n(0)} = \frac{\delta\nu(eV)}{\nu(0)}.$$

Fluctuation Pseudogap in N(S)-I-N Junction (Varlamov, Dorin, 1983)

$$I_T(V) = -e \ \mathrm{Im}\ K^R \ (\omega_\nu \to -ieV) \ ,$$

$$K \ (\omega_\nu) = 4T \sum_{\varepsilon_n} \sum_{\mathbf{p},\mathbf{k}} |T_{\mathbf{p},\mathbf{k}}|^2 G_R \ (\mathbf{p}, \ \varepsilon_n + \omega_\nu) \ G_L \ (\mathbf{k}, \ \varepsilon_n)$$

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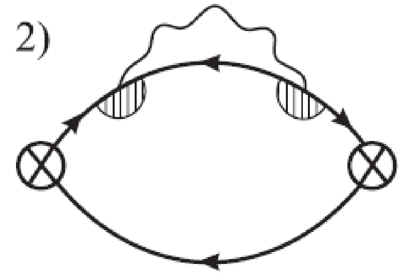
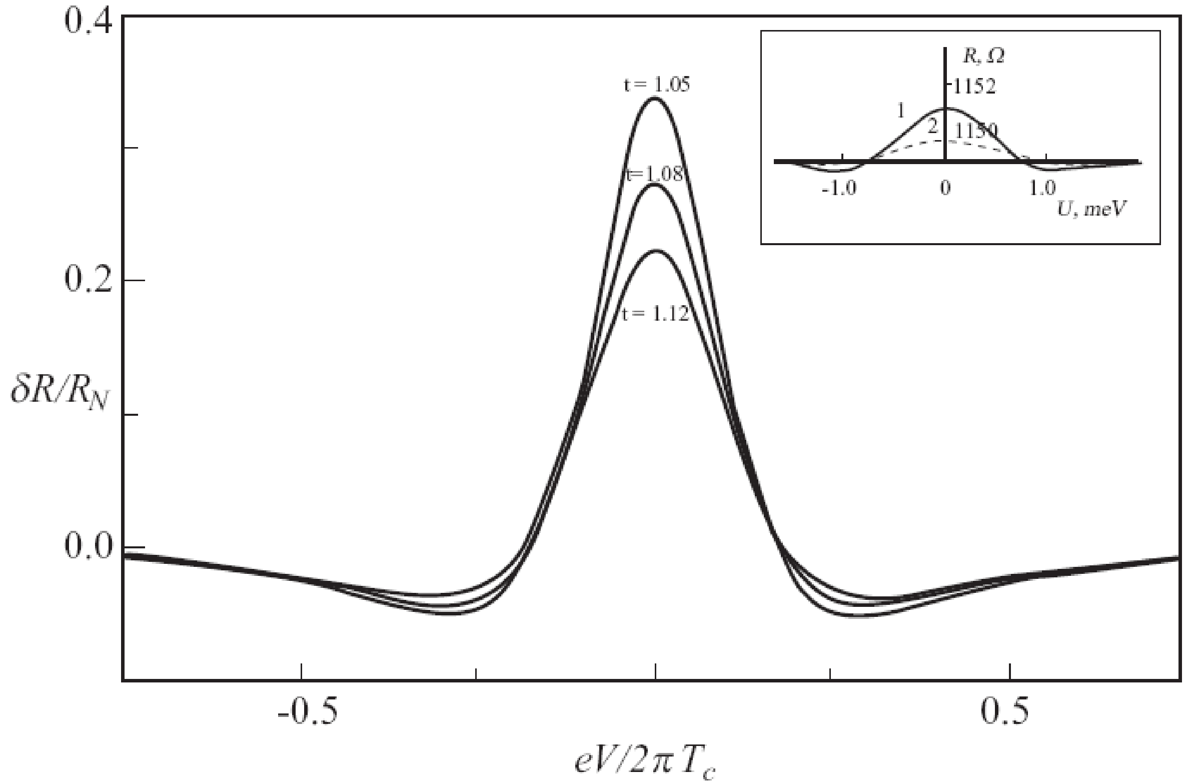
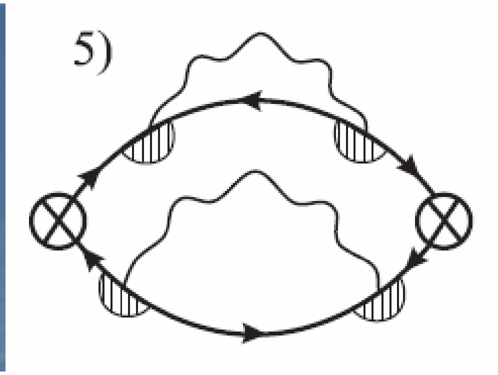
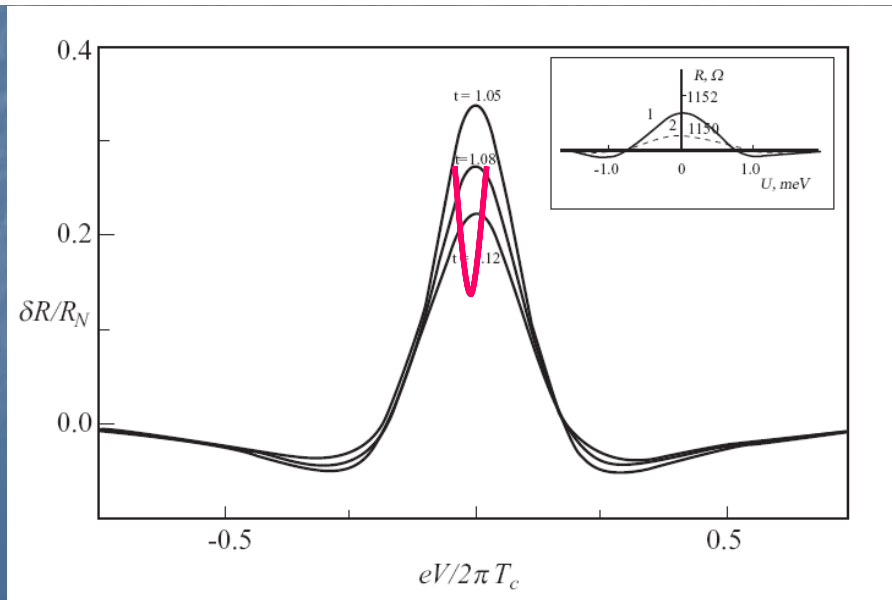


$$G^{(1)}_{qp}(V) = \frac{Gi_{(2d)}}{7\zeta\ (3)\ R_n}\ \left(\ln\frac{1}{\epsilon}\right)\ \mathrm{Re}\ \psi''\ \left(\frac{1}{2} - \frac{ieV}{2\pi T}\right)$$





$$\delta G_{\mathrm{fl}}^{(2d)}(0,\,\epsilon) \sim \int_{-\infty}^{\infty} \frac{dE}{\cosh^2\left(\frac{E}{2T}\right)} \left[\delta\nu_{(2)}^{(d)}(E,\,\epsilon)\right]^2 \sim \frac{Gi_{(2d)}^2}{\epsilon^3}.$$



Fluctuation Phenomena in N(S)-I-S Junction

Quasipartical current

$$K\ (\omega_\nu) = T \sum_{\varepsilon_n} \sum_{\mathbf{p}_1,\mathbf{k}_1} \sum_{\mathbf{p}_2,\mathbf{k}_2} T_{\mathbf{k}_1\mathbf{p}_1} G_n(\mathbf{p}_1,\ \mathbf{p}_2,\ \varepsilon_n + \omega_\nu) T^*_{\mathbf{p}_2\mathbf{k}_2} G_s(\mathbf{k}_2,\ \mathbf{k}_1,\ \varepsilon_n).$$

$$I_{qp}(V) = \frac{2C_{NN}}{e} \Delta_2 \sum_{n=1}^{\infty} (-1)^{n+1}\ K_1\ \left(\frac{n\Delta_2}{T}\right) \sinh\left(\frac{neV}{T}\right)$$

At low temperatures is exponentially small

Current related with Cooper pairs tunneling

Boundary Hamiltonian

$$\widehat{\mathcal{H}}_b = \frac{1}{8e^2 R_n} \int \exp(i\mathbf{qr}) d^2\mathbf{r} \times$$

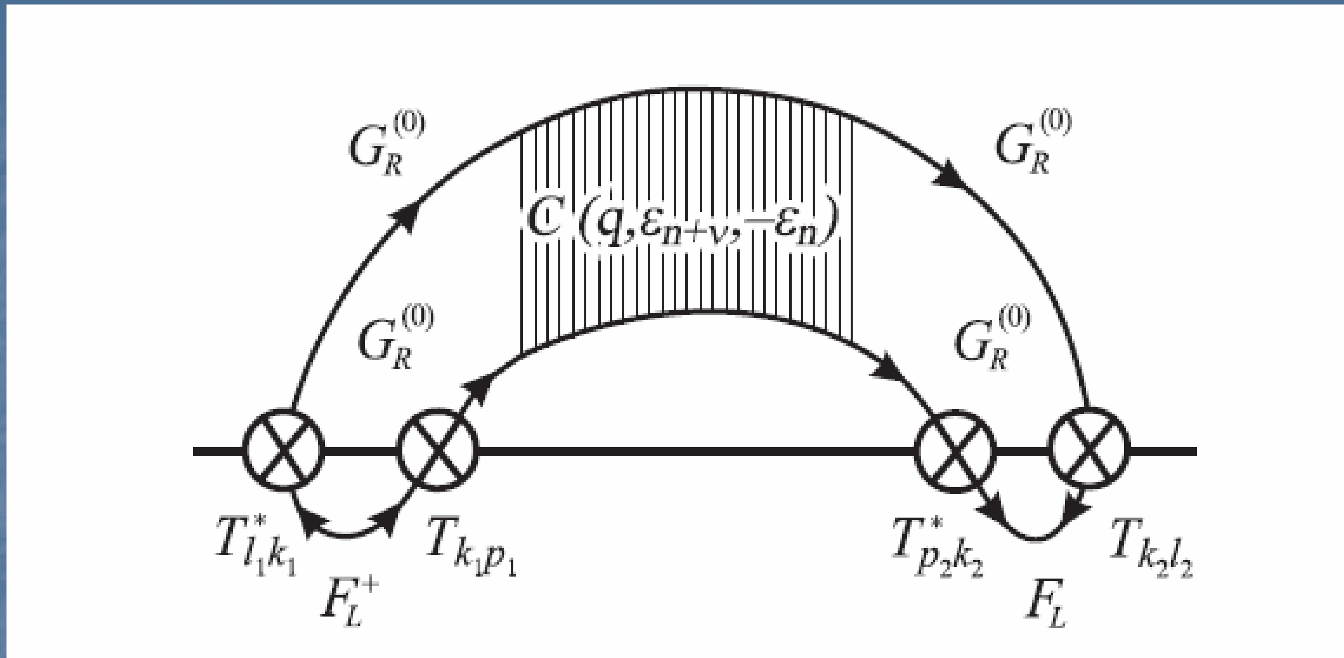
$$\times \sum_{\mathbf{q}} \left(e^{-i\varphi(\mathbf{r})} \widehat{a}_{\mathbf{p}+\mathbf{q}/2\uparrow} \widehat{a}_{\mathbf{p}-\mathbf{q}/2\downarrow} + e^{i\varphi(\mathbf{r})} \widehat{a}^{+}_{\mathbf{p}+\mathbf{q}/2\uparrow} \widehat{a}^{+}_{\mathbf{p}-\mathbf{q}/2\downarrow} \right)$$

$$I_b = e \int_{-\infty}^{0} \left\langle \left[\widehat{\mathcal{H}}_b(t), \left[\sum_{\mathbf{p},\sigma=\pm 1/2} \widehat{a}^{+}_{\mathbf{p}\sigma} \widehat{a}_{\mathbf{p}\sigma}, \widehat{\mathcal{H}}_b\ (0) \right] \right] dt \right\rangle$$

$$I_b\ (V, H) = -\frac{e}{\left(4\nu e^2 R_n\right)^2} \operatorname{Im} \sum_{\mathbf{p},\mathbf{p}'} \mathcal{L}^R(\mathbf{p}, \mathbf{p}', \mathbf{q}, 2eV).$$

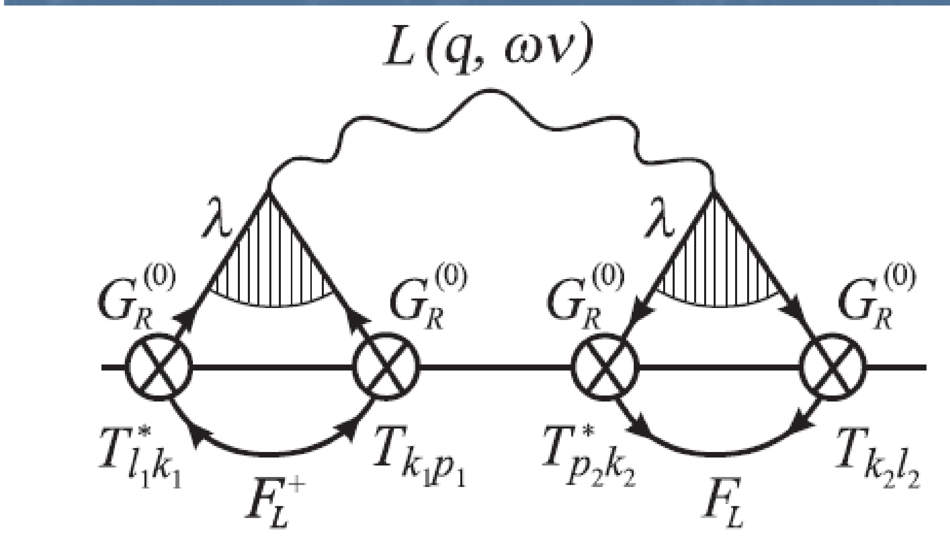
$$\begin{aligned} L^R\ (\mathbf{q}, 2eV) &= \sum_{\mathbf{p},\mathbf{p}'} \mathcal{L}^R(\mathbf{p}, \mathbf{p}', \mathbf{q}, 2eV) \propto \frac{\Pi^R}{1 - g\Pi^R} \\ &= \Pi^R + \Pi^R \frac{1}{1/g - \Pi^R} \Pi^R. \end{aligned}$$

Andreev's conductance (Volkov et al, 1993)

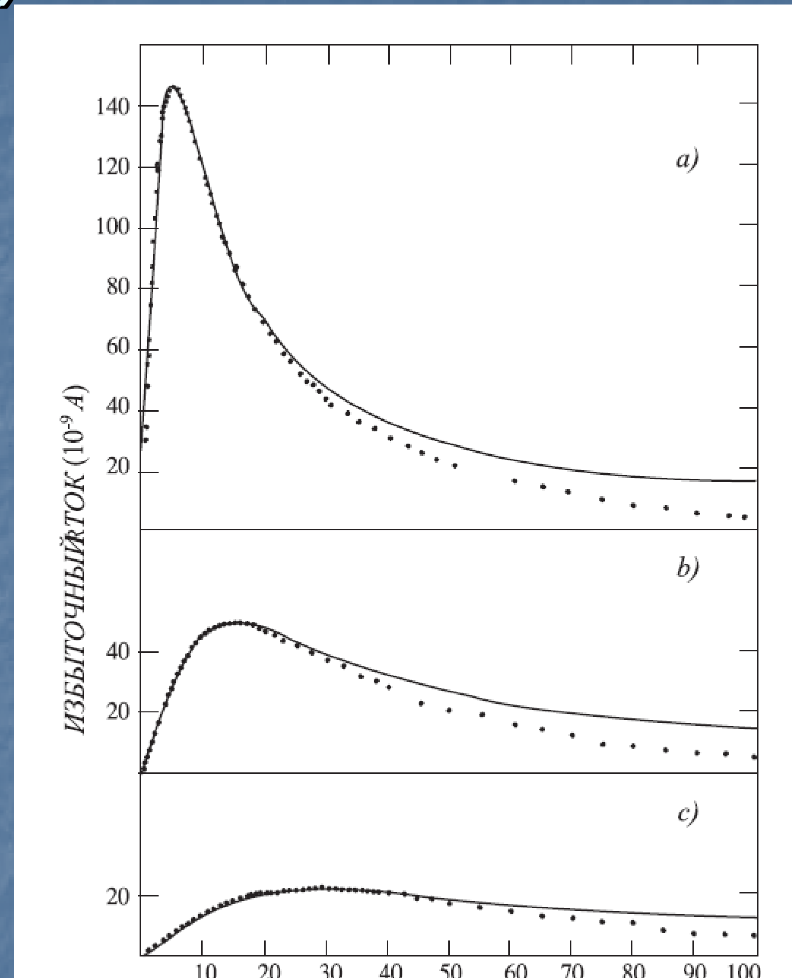


$$\mathrm{Im}\ \Pi\ (2eV) = -\nu\ \mathrm{Im}\ \psi\ \left(\frac{1}{2} + \frac{-2ieV + \mathcal{D}q^2}{4\pi T}\right)$$

Superconducting "pair field" susceptibility (Scalapino, 1970)

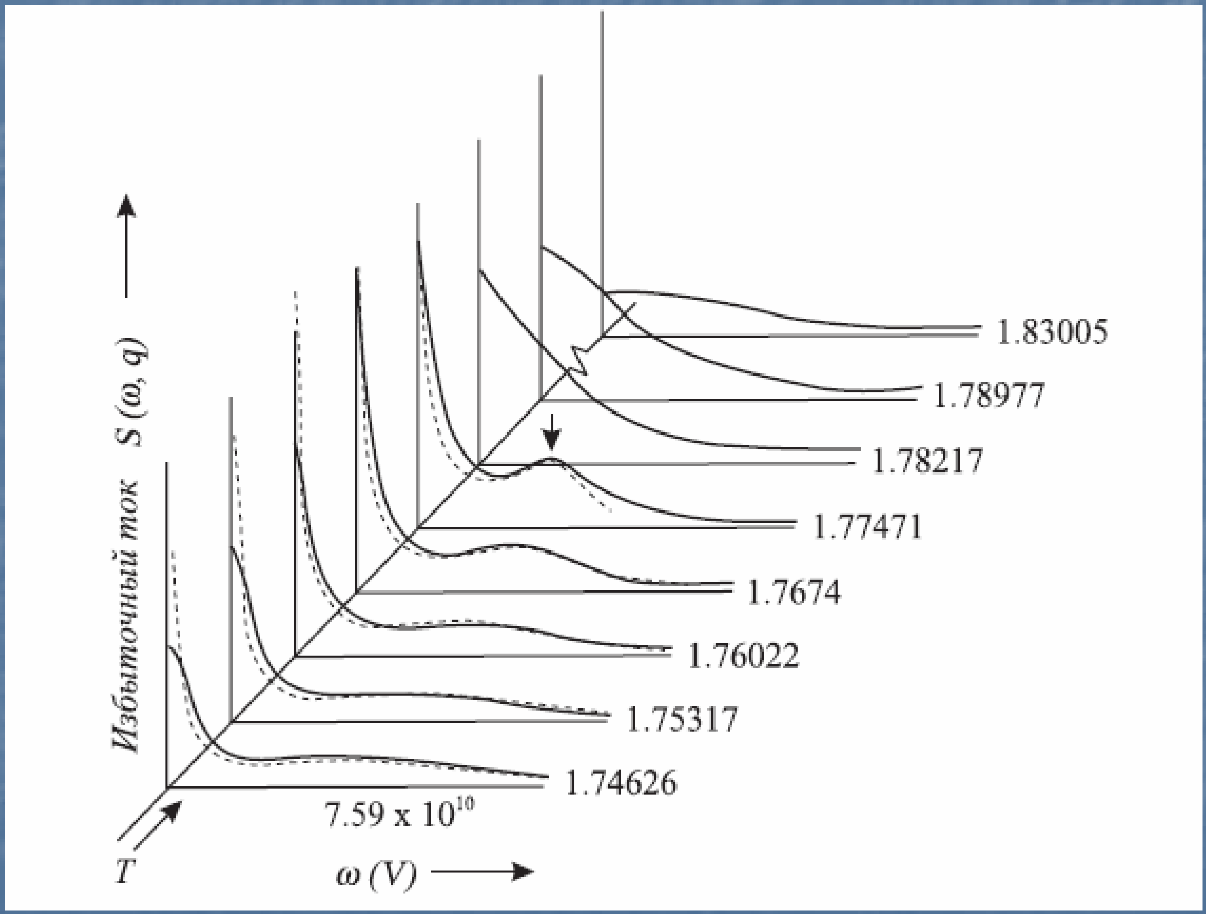


$$\omega = 2eV \text{ и } q = 2eH\lambda_L.$$



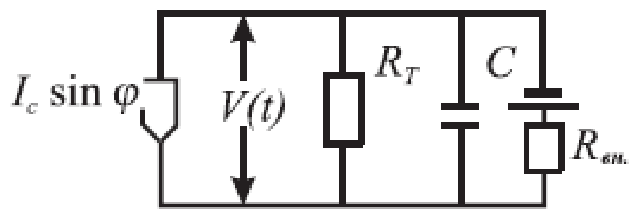
$$I_{\text{fl}}\,(q,\,\omega) = \frac{S}{16e^3R_n^2d\epsilon}\,\ln^2\,\frac{\Delta_2}{T}\,\frac{\omega\tau_{\text{GL}}}{\left(1+\xi^2q^2\right)^2+\omega^2\tau_{\text{GL}}^2},$$

Collective modes in SC (Goldman et al, 1973)



Fluctuations in Josephson structures

Fluctuation smearing of the emission line (Larkin, Ovchinnikov, 1967)



$$Z^{-1}(\omega) = -i\omega C + R_T^{-1} + R_{ext}^{-1}.$$

$$Q = \Gamma R^* C.$$

$$K(\omega) = \frac{2}{\pi}\int_{-\infty}^{\infty} e^{-i\omega\tau}\,\overline{I(t)I(t+\tau)}\,d\tau.$$

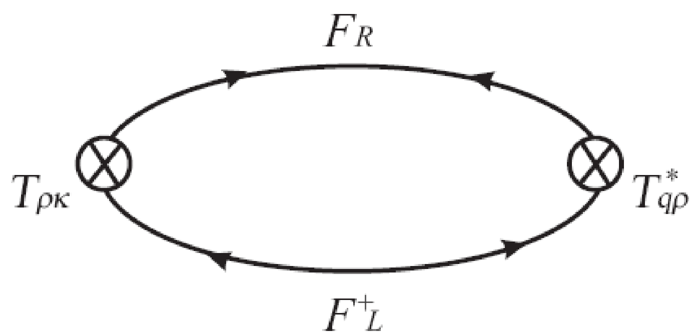
$$K(\omega) \sim \frac{1}{\pi}\frac{\Gamma}{(\omega-\omega_J)^2+\Gamma^2}.$$

$$Q \ll 1,$$

$$K(\omega) \sim \frac{1}{\sqrt{\pi}\tilde{\Gamma}}\exp\left[-\left(\frac{\omega-\omega_J}{\tilde{\Gamma}}\right)^2\right], \quad \tilde{\Gamma} = 2e\sqrt{\frac{2T^*}{C}}$$

$$Q \gg 1$$

Thermal fluctuations of the order parameter



$$I_c = 4e \sum_{\mathbf{p},\mathbf{q}} |T_{\mathbf{pq}}|^2 \; T \sum_{\varepsilon_n} F_L^+ \; (p,\, \varepsilon_n) \; F_R \; (q,\, -\varepsilon_n) \; ,$$

$$I_c = \frac{\pi}{eR_n} T \sum_{\varepsilon_n} \frac{\Delta_L}{\sqrt{\Delta_L^2 + \varepsilon_n^2}} \frac{\Delta_R}{\sqrt{\Delta_R^2 + \varepsilon_n^2}},$$

$$I_c = \frac{\pi \Delta(T)}{2eR_n} \tanh \frac{\Delta \; (T)}{2T}.$$

Asymmetric junction (Larkin, Varlamov, 2004)

$$I_c(t) = \frac{\pi}{eR} T \sum_n \int d^2\mathbf{r} \frac{\Delta_1^{(\mathrm{fl})}(\mathbf{r},\, t)}{|\varepsilon_n|} \frac{\Delta_2}{\sqrt{\Delta_2^2 + \varepsilon_n^2}}$$

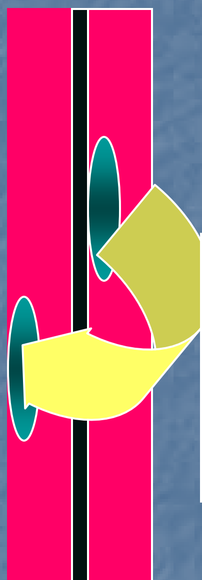
$$K(\omega) = \frac{2}{\pi} F_c^2 \left(\frac{\Delta_2(T_{c1})}{2\pi T_{c1}} \right) \int d^2\mathbf{r}\, d^2\mathbf{r}' \times$$

$$\times \int_{-\infty}^{\infty} e^{-i(\omega-\omega_J)\tau} \overline{\langle \Delta^{(\mathrm{fl})}(\mathbf{r},\, t)\, \Delta^{*(fl)}(\mathbf{r}',\, t+\tau) \rangle} d\tau =$$

$$\mathrm{Re}\, K(\omega) = \frac{64 T_{c1}^2 S}{\pi^2 \nu e^2 R^2} \left(\ln^2 \frac{\Delta_2(T_{c1})}{\pi T_{c1}} \right) \frac{\Gamma(\epsilon)}{\Gamma^2(\epsilon) + (\omega - \omega_J)^2}$$

Symmetric junction (small junction, Kulik, 1970; large junction, Larkin, Varlamov, 2004)

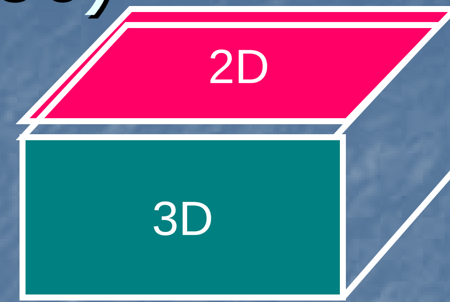
$$I_c(t) = \frac{\pi}{eR} T \sum_n \int d^2\mathbf{r} \frac{\Delta_1^{(\mathrm{fl})}(\mathbf{r}, t)}{\sqrt{\Delta_1^{(fl)2} + \varepsilon_n^2}} \frac{\Delta_2^{(\mathrm{fl})}(\mathbf{r}, t)}{\sqrt{\Delta_2^{(fl)2} + \varepsilon_n^2}}$$



$$K(\omega) = \frac{\pi S}{2e^2R^2T^2} \left[\frac{1}{4m\nu\xi^2}\right]^2 \int_{-\infty}^{\infty} e^{-i(\omega-\omega_J)\tau} \int d^2\mathbf{q} \, \langle \Psi\Psi^* \rangle_{t,q\to 0}^2 \, d\tau$$

$$\mathrm{Re}\, K(\omega) = \frac{16\pi^3 TS}{7\zeta(3)\, e^2R^2\nu} Gi_{(2)} \ln \frac{32T\Gamma(\epsilon)/\pi}{4\Gamma^2(\epsilon) + (\omega-\omega_J)^2}$$

Fluctuation renormalization of the Josephson current (Varlamov, Dorin, 1986)



$$I_c = \frac{\pi \Delta^2(T)}{4eTR_n}.$$

Mean field in the vicinity of T_c gives:

$$\Delta^2(T) \rightarrow \Delta_0^2(T) = \frac{8\pi^2 T_{c0}^2}{7\zeta(3)} |\epsilon_0|,$$

$$I_c^{(0)}(|\epsilon_0|) = \frac{2\pi^3 T_{c0}}{7\zeta(3) e R_n} |\epsilon_0|$$

where $|\epsilon_0| = (T_{c0} - T)/T_{c0}$

GL fluctuation theory gives:

$$\Delta^2(T) \rightarrow \langle \Delta(T) \rangle^2.$$

$$\langle \Delta(T) \rangle^2_{(\mathrm{fl})} = \Delta_0^2 - \frac{1}{4mC_{(D)}} \left[3 \langle \psi_r^2 \rangle + \langle \psi_i^2 \rangle \right]$$

$$I_c(T) = \frac{\pi \langle \Delta(T) \rangle^2_{(\mathrm{fl})}}{4eTR_n} = I_c^{(0)}(|\epsilon_0|) + \delta I_c^{(\mathrm{fl})}(|\epsilon_0|).$$

$$\delta I_{c(2)}(|\epsilon|) = \frac{4\pi^3 T_c}{7\zeta(3) eR_n} Gi_{(2)} \left[\ln \frac{|\epsilon|}{Gi_{(2)}} + \ln \left(\frac{\xi(T)}{L_J} \right) \right].$$

Exponential tail in Josephson junction close to T_c (Larkin, Varlamov, 2004)

$$\mathcal{F}[\varphi] = \frac{n_s}{4m} \int d^2\mathbf{r} \left[\nabla \varphi(\mathbf{r})\right]^2 = \frac{n_s}{4m} \sum_{\mathbf{k}} \mathbf{k}^2 \varphi_{\mathbf{k}}^2 .$$

+

$$\delta E_J^{(\mathrm{fl})}(\epsilon) = \frac{E_J}{2S} \int d^2\mathbf{r} \left[\varphi^{(\mathrm{fl})}(\mathbf{r})\right]^2 .$$

$$F\left(\varphi^{(\mathrm{fl})}\right) = \frac{n_s}{2m} \sum_{\mathbf{k}} \left(\mathbf{k}^2 + \frac{1}{L_J^2}\right) \varphi_{\mathbf{k}}^2 ,$$

where the Josephson length L_J is determined by relation

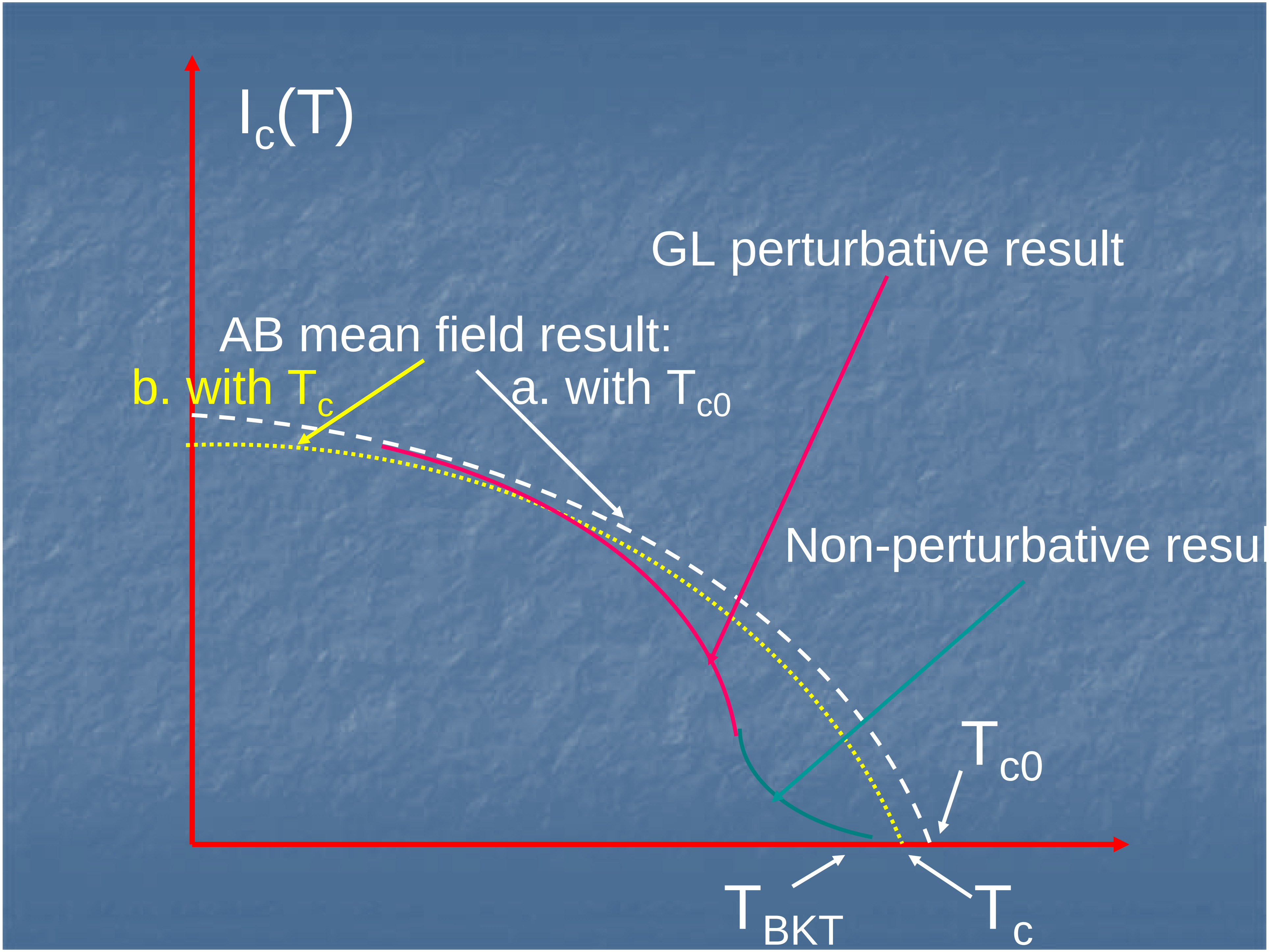
$$\frac{n_s}{2mL_J^2} = \frac{E_J}{2S} .$$

$$I_{\mathrm{c}}(\varphi) = \frac{E_J}{2S}\left\langle \sin\left(\varphi + \varphi^{(\mathrm{fl})}\right)\right\rangle$$

$$\delta E_J^{(\mathrm{fl})}(\epsilon) = E_{J0}\exp\left(-\frac{2Gi_{(2d)}}{|\epsilon|}\ln\frac{L_J}{\xi(T)}\right)$$

$$I_{\mathrm{c}}(\epsilon) = I_{\mathrm{c}}^{(AB)}(\epsilon)\exp\left(-\frac{\epsilon^*}{|\epsilon|}\right)$$

$$\epsilon^* = Gi_{(2d)}\ln\left[Gi_{(2d)}\left(\frac{L_J}{\xi}\right)^2\right]$$



$I_c(T)$
GL perturbative result
AB mean field result:
b. with T_c
a. with T_{c0}
Non-perturbative resul
T_{c0}
T_{BKT}
T_c

Fluctuation spectroscopy of granularity in superconducting structures

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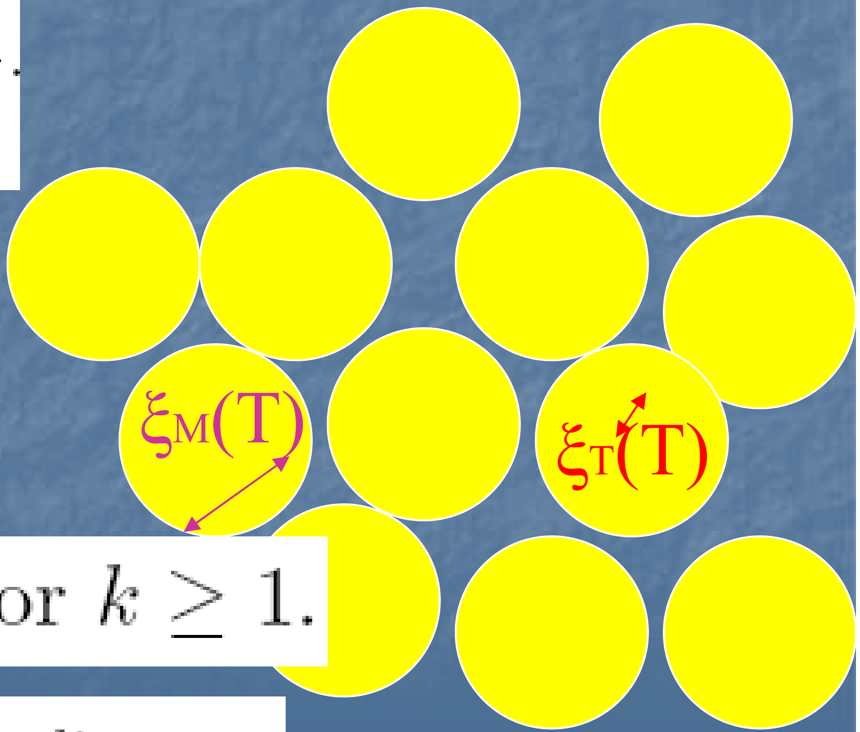
(Dated: June 6, 2007)

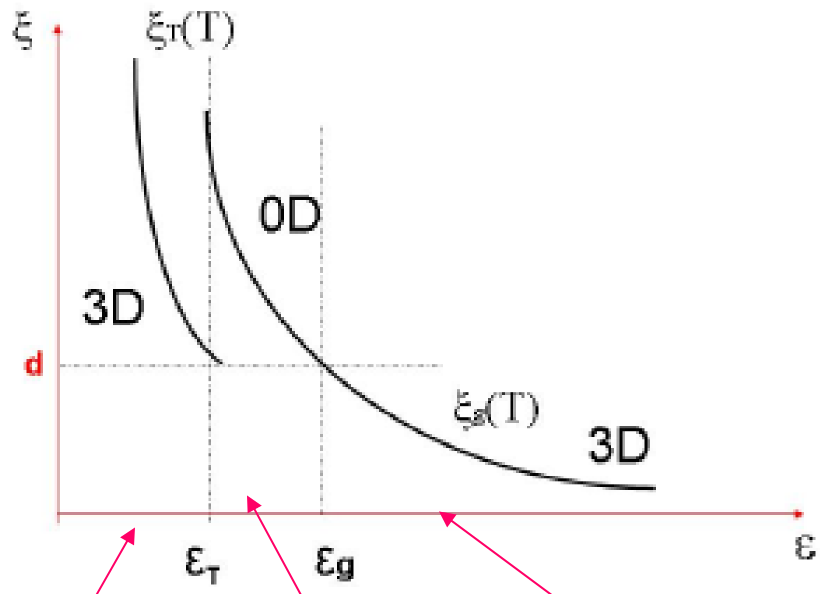
$$\mathcal{L}\left(q_k,\omega_m\right) = -\frac{1}{\nu}\frac{1}{\epsilon - \mathrm{i}\pi\omega_m/8T_\mathrm{c} + \xi_\mathrm{g}^2 q_k^2}.$$

$$q_k d \;=\; 2\tan\left(q_k d/2\right)$$



$$q_0 = 0 \text{ and } q_k d \approx 2\pi k + \pi \text{ for } k \geq 1.$$

$\Gamma = g_\mathrm{T}\delta$ is the intergrain tunneling rate

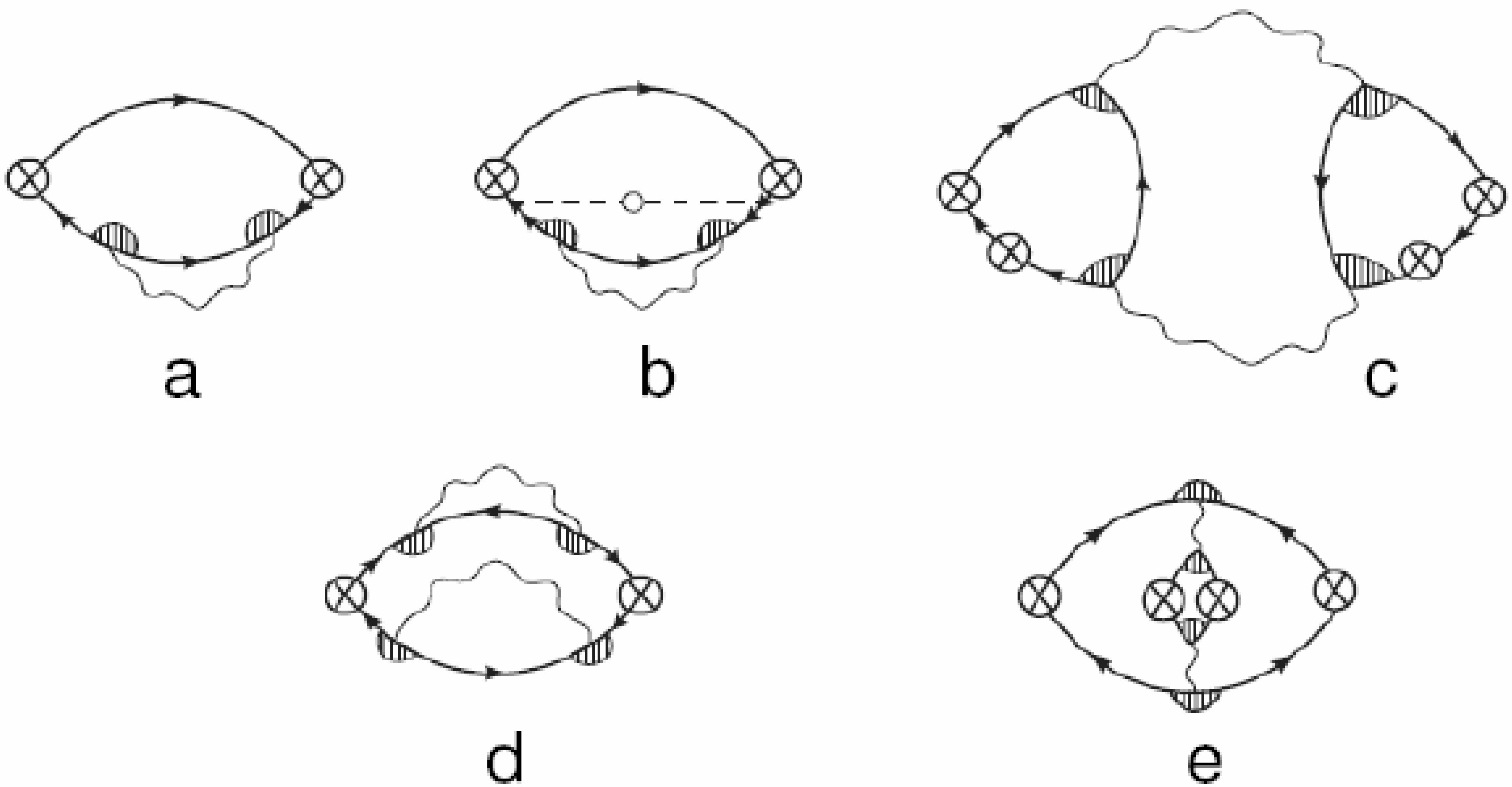


$$\xi_{\mathrm{g}}(\varepsilon) \equiv \frac{\xi_{\mathrm{g}}}{\sqrt{\varepsilon}}\,, \xi_{\mathrm{T}}(\varepsilon) \equiv \frac{\xi_{\mathrm{T}}}{\sqrt{\varepsilon}}\,.$$

$$\varepsilon \lesssim \varepsilon_{\mathrm{T}}\,,$$

$$(\xi_{\mathrm{g}}/d)^2 \equiv \varepsilon_{\mathrm{g}} \lesssim \varepsilon \lesssim 1\,,$$

$$(\Gamma/T_c) \equiv \varepsilon_{\mathrm{T}} \lesssim \varepsilon \lesssim \varepsilon_{\mathrm{g}}$$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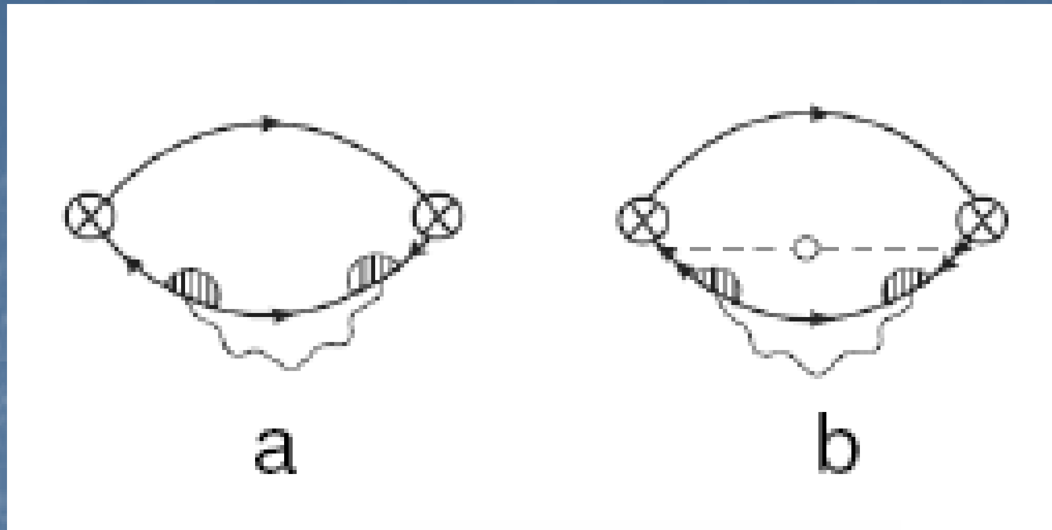
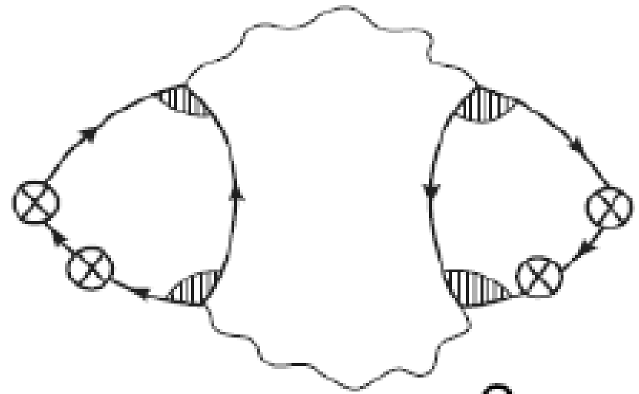


$$\sigma_{\rm DoS}(\epsilon) \sim -\frac{e^2}{d}\left(\frac{\Gamma}{T_{\rm c}}\right)\sum_{\mathbf{q}}\frac{1}{\epsilon+\xi_{\rm g}^2 q_k^2}.$$

$$\sigma_{\rm DoS}(\epsilon) \sim -\frac{e^2}{d} \times \begin{cases} \dfrac{\epsilon_{\rm T}}{\sqrt{\epsilon_{\rm g}\epsilon}}, & \epsilon_{\rm g} \lesssim \epsilon \lesssim 1 \\ \dfrac{\epsilon_{\rm T}}{\epsilon}, & \epsilon_{\rm T} \lesssim \epsilon \lesssim \epsilon_{\rm g} \end{cases}$$

$$\epsilon_{\rm T} \equiv \Gamma/T_{\rm c} \ll 1$$



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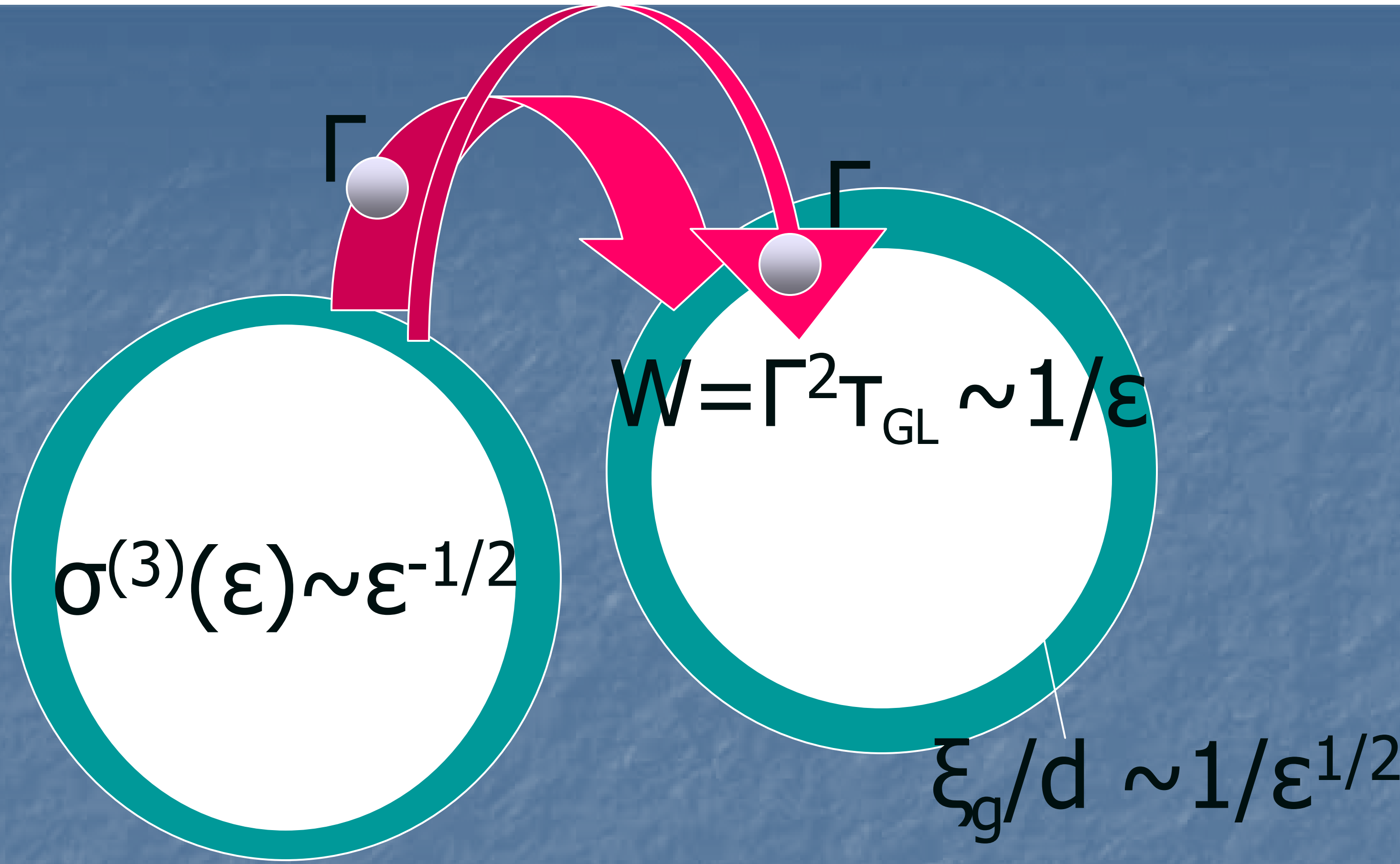
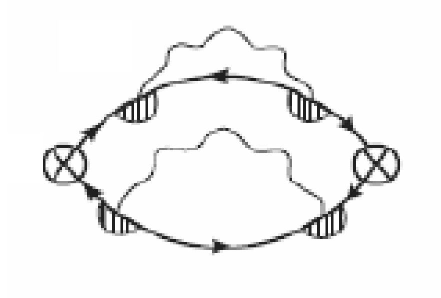
$$\sigma_{\mathrm{AL}}(\varepsilon) \sim \frac{\pi e^2}{d}\left(\frac{\Gamma}{T_{\mathrm{c}}}\right)^2 \sum_{k,l} \frac{1}{\left(2\varepsilon+\xi_{\mathrm{g}}^2 q_k^2+\xi_{\mathrm{g}}^2 q_l^2\right)\left(\varepsilon+\xi_{\mathrm{g}}^2 q_k^2\right)\left(\varepsilon+\xi_{\mathrm{g}}^2 q_l^2\right)}$$

$$\sigma_{\mathrm{AL}}(\epsilon) \sim \frac{e^2}{d}\left(\frac{\Gamma}{T_{\mathrm{c}}}\right)^2 \int_{-\infty}^{\infty} \mathrm{d}\zeta \left(\sum_{\mathbf{q}_k} \frac{1}{\left(\epsilon+\xi_{\mathrm{g}}^2 q_k^2\right)^2+\zeta^2}\right)^2$$

$$\sim \frac{e^2}{d} \times \begin{cases} \dfrac{\epsilon_{\mathrm{T}}^2}{\epsilon_{\mathrm{g}}\epsilon^2}, & \epsilon_{\mathrm{g}} \lesssim \epsilon \lesssim 1 \\ \dfrac{\epsilon_{\mathrm{T}}^2}{\epsilon^3}, & \epsilon_{\mathrm{T}} \lesssim \epsilon \lesssim \epsilon_{\mathrm{g}}. \end{cases}$$



$$\sigma^{(3t)}(\varepsilon) = \sigma^{(3)}(\varepsilon)\,\Gamma^2\,\tau_{GL}(\varepsilon)\,\xi g(\varepsilon)/d \sim 1/\varepsilon^2$$



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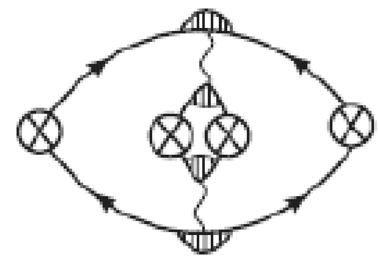

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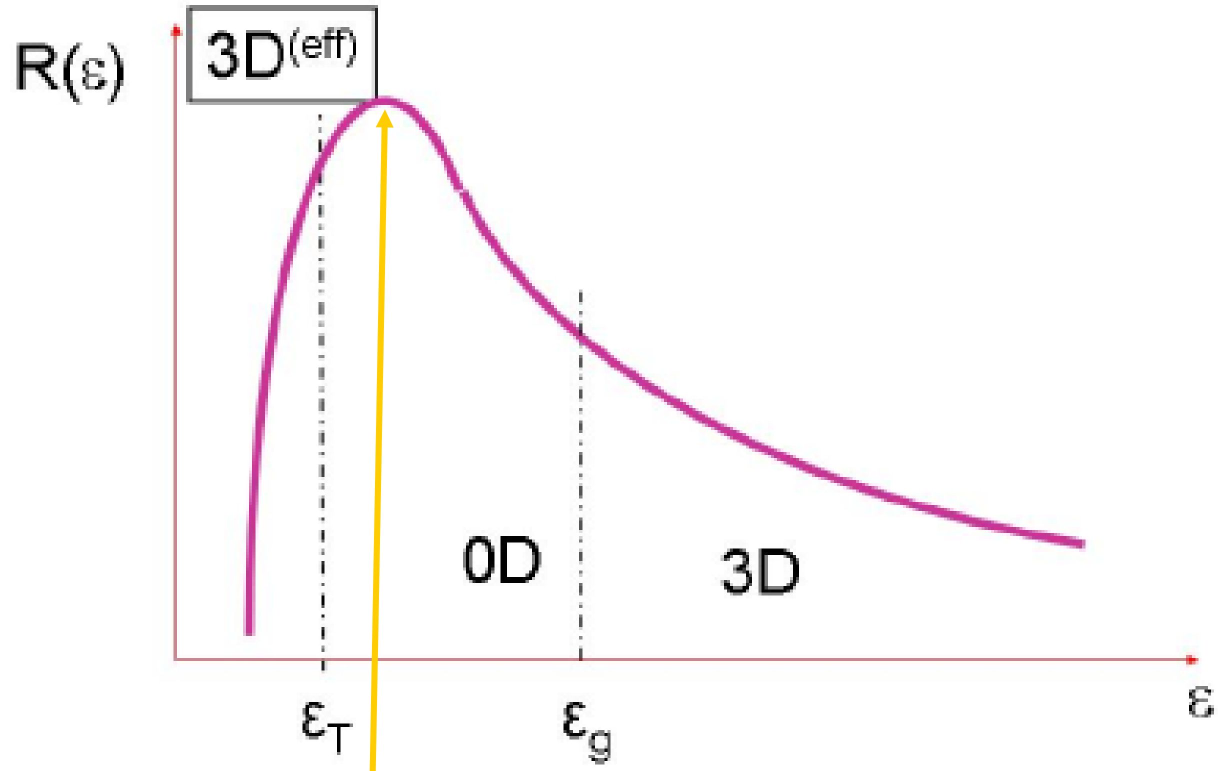
$$\sigma_{\mathrm{MT}} \sim \frac{e^2}{d}\left(\frac{\delta}{T}\right)\left(\frac{\Gamma}{T_{\mathrm{c}}}\right)\sum_{\mathbf{q}_k}\sum_{\mathbf{q}_l}\frac{1}{(\epsilon+\xi_{\mathrm{g}}^2 q_k^2)} \times \frac{1}{(\epsilon+\xi_{\mathrm{g}}^2 q_l^2)}\frac{1}{\left[2\gamma_\varphi \;+\xi_{\mathrm{g}}^2 q_k^2+\xi_{\mathrm{g}}^2 q_l^2\right]^3},$$

$$\sigma_{\mathrm{MT}}(\epsilon) \sim \frac{e^2}{d}\left(\frac{\delta}{T}\right)\frac{\epsilon_{\mathrm{T}}}{\gamma_\varphi^3}\frac{1}{\epsilon^2}\times\begin{cases}\dfrac{\gamma_\varphi}{\epsilon_{\mathrm{g}}}, & \epsilon_{\mathrm{g}} \lesssim \gamma_\varphi \lesssim \epsilon \lesssim 1\\ \dfrac{\epsilon}{\epsilon_{\mathrm{g}}}, & \epsilon_{\mathrm{g}} \lesssim \epsilon \lesssim \gamma_\varphi \lesssim 1\\ 1, & \epsilon_{\mathrm{T}} \lesssim \epsilon \lesssim \min\left(\epsilon_{\mathrm{g}}, \gamma_\varphi\right)\end{cases}$$

$$\Gamma/T >> (\delta/T)^{1/3}$$



$$\varepsilon_{max} = (\Gamma/T)^{1/3}$$