Vortex Lattices in Tilted Magnetic field

Collaborations:
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Motivation:
Paramagnetic Meissner effect in polycrystalline HTSC: Spontaneous supercurrents due to random 0-π frustrations?
Alternative mechanism based on flux compression
Lattice of Josephson vortices

\[ \frac{c_y}{c_z} = \gamma \]

Small fields: Dilute lattice

High fields \( B_x > B_{cr} = \Phi_0/2\pi s\lambda_J \) (~ 0.5 T) : Dense lattice

Bulaevskii & Clem, 1991

Core size \( \lambda_J = \gamma s \)

Bi\(_2\)Sr\(_2\)CaCu\(_2\)O\(_{8+\delta}\) (BSCCO)

Pancake stacks


\[ B(x) \]

London penetration depth \( \lambda \approx 200 \text{ nm} \)

\( \text{CuO}_2\)-bilayers

Pancake-vortex stacks
Magnetic and Josephson coupling

**Magnetic coupling:**
- nonlocal: range $\lambda$
- weaker than in-plane interaction by $s/\lambda$

**Magnetic cage:**

**Josephson coupling:**
phase mismatch between neighboring layers

$-E_J \cos(\phi_1-\phi_2)$

**Relative strength:**
Parameter $\alpha = \lambda/\gamma s$

**BSCCO:**
- $\gamma = 200 - 700$
- $\lambda/s = 130 - 250$
- $\alpha = 0.3-0.9$
Interaction between Jos. vortex and pancake stacks

Crossing configuration

Deformation energy

\[ E = \sum_n \left( -\frac{s\Phi_0}{c} j_n(0) u_n + \frac{Ku_n^2}{2} \right) \]

Jos. Vortex current

Magnetic cage

Crossing energy

\[ E_x = -\frac{\Phi_0^2}{2\pi^2\gamma^2s \ln(3.5\gamma s / \lambda)} \]

~ 200 K

Maximum displacement:

\[ u_1 = \frac{\lambda^2}{\gamma s} \]

~ 100 nm

Numerics: instability at \( \alpha = \frac{\lambda}{\gamma s} \approx 0.69 \)

Penetration of c-axis field in presence of in-plane field
stacks vs kinks

\[ \alpha = \frac{\lambda}{\gamma} s = 0.45 - 0.5 \]

- \( \alpha < 0.3 \)
- \( \alpha > 0.7, \gamma \gg 1 \)

Bulaevskii et al. 1992; Huse 1992; AEK, 1999…
Feinberg et al. 1990; Ivlev et al. 1991; Bulaevskii et al. 1992…
Visualization

Decorations:  Bolle et al., PRL, 1991
             Grigorieva et al., PRB, 1995
             Tokunaga et al., PR B, 2003
             (Tamegai Group)

Magnetooptics:
             Vlasko-Vlasov et al., PR B 2002
             Tokunaga et al., PR B, 2002

Also:

Lorentz microscopy:
             Matsuda et. al.(Tonomura group), Science, 2001

Scanning Hall Probe:
             Grigorenko et. al. (Bending group), Nature, 2001
Outline

• Vortex-chain states and transitions
• Josephson vortex inside pancake-vortex lattice
• Phase diagram in tilted fields at intermediate anisotropies
Lawrence-Doniach model in London limit

Regular phase $\phi_n(x,y) + \text{pancake displacements } u_n$ (vortex phase $\phi_{v,n}$)

Energy (distances, $x,y < \lambda_c$, $z < \lambda_{ab} \rightarrow \text{no screening}$):

$$E[\phi_n, u_n] = \sum_n \int d^2r \left[ \frac{J}{2} (\nabla \phi_n)^2 + E_J \left( 1 - \cos \left( \phi_{n+1} - \phi_n + \phi_{v,n+1} - \phi_{v,n} - \frac{2\pi s}{\Phi_0} B_{xy} \right) \right) \right]$$

$$+ \frac{1}{2} \sum_{n,m} U(u_n - u_m)$$

Full 3D model was used to study isolated chain

2D model (pancake vortex lattice, at $a < \gamma s$):

1. $\phi_n(x,y) = \bar{\phi}_n(y) + \tilde{\phi}_n(x,y)$
2. $\tilde{\phi}_n(x,y) \approx \bar{\phi}_{v,n}(x,y)$
3. $F_{J,loc}[\bar{\phi}_n, u_n] = E_J \int dx dy \left[ \cos(\bar{\phi}_{n+1} - \bar{\phi}_n + \bar{\phi}_{n+1} - \bar{\phi}_n) - \cos(\bar{\phi}_{n+1} - \bar{\phi}_n) \right]$
Vortex Chains in tilted fields


Dilute Josephson vortex lattice + very small c-axis field → Vortex Chains

key parameter $\alpha = \lambda / \gamma s$

Really high anisotropy $\alpha < 0.3$ vs not-so-high anisotropy $\alpha > 0.7$

Crossing Chains

Tilted Chains

Competition between Magnetic coupling and interaction with JVs

(likes straight stacks) (likes tilted stacks)

two periods: $a$ and $N = 2N_l$
Attraction between deformed stacks

A. Buzdin and I. Baladié, PRL 2002

Interaction energy for $a < \gamma s$

$$U_{\text{int}}(a) = 2\varepsilon_0 \left[ K_0\left(\frac{a}{\lambda}\right) - \frac{\langle u^2 \rangle}{a^2} \right]$$

$$\propto \exp\left(-\frac{a}{\lambda}\right)$$

Minimum of $U_{\text{int}}(a)$ at $a_m = \lambda \ln \frac{C\lambda^2}{\langle u^2 \rangle}$

Valid only if $a_m < \gamma s \rightarrow$ not practical for BSCCO

In general: $\min_a [U(a)]$

Energy per unit cell

Also: attraction between tilted stacks

1\textsuperscript{st} scenario, \(0.4 < \alpha < 0.55\)

Locked $\rightarrow$ Crossing Chain $\rightarrow$ Tilted Chain

\(\alpha = 0.5\)
2\textsuperscript{nd} scenario $0.55 < \alpha < 0.65$

Locked $\rightarrow$ Kinked $\rightarrow$ Crossing Chain $\rightarrow$ Tilted Chain

1. kink penetration
2. 1\textsuperscript{st} order transition due to competing interaction energies

![Graph showing energy per pancake and density of pancakes/kinks]
Typical phase diagram for $0.5 < \alpha < 0.65$
Experimental observation of transition

A. Grigorenko et. al. (S. Bending Group), Nature, 2001

Transition to tilted chains

“… as $H_c$ is increased further, 'crystallites' of well resolved PV stacks, with ten times higher flux density, nucleate and grow ($H_c = 4\text{Oe and } 4.4 \text{ Oe}$).”
Vortex Chains: practical application

Physics Today: Advertisement of Scanning Probe Microscope
In-plane vortex inside dense pancake lattice

\[ B_z = 10 \text{ – } 100 \text{ G} \]

Crossover Josephson vortex \( \rightarrow \) soliton

Renormalization of Josephson vortex by pancakes

$$\alpha = \lambda / \gamma s \ll 1$$

Typical field

$$B_\lambda = \frac{\Phi_0}{4\pi\lambda^2} \ln \frac{a}{r_w} \sim 30 \text{ - } 70 \text{ G}$$

Vortex energy

$$\varepsilon_J = \sqrt{\frac{B_\lambda}{B_z + B_\lambda}} \varepsilon_{J0}$$

Core size

$$\lambda_J = \sqrt{\frac{B_\lambda}{B_z + B_\lambda}} \gamma s$$

Dense Lattice, $B_z > B_\lambda$:  

Maximum displacement $u_1/a \approx 0.5 \alpha$

Number of rows in the core $\approx 1/\alpha$
Solitonlike cores at $\alpha > 0.5$

$$B = \Phi_0/\gamma^2$$

Displacements in the central row

Phase difference between two central layers

$\lambda/\gamma$ vs $\lambda/\gamma_s$

- $0.3$
- $0.5$

$\sim \lambda/\gamma$
Soliton lattice with increasing $B_x$
Composite Lattices and Tilted Lattice at large tilt angles

Phase diagram at moderate $\alpha$
Composite Lattices


- Dense-lattice case $a < \lambda$
- Tilt angle of field 1 $<< \tan \Theta << \gamma$

For fixed field: two independent parameters: $M$ and $r = a/a_t$

$$E = \min_{M,r} [E(M,r)]$$
Why composite lattice may be better?

Energy per tilted row

\[ F_J = \frac{K_J}{2} \left[ \tan(\theta) \right]^2 \frac{c_y}{b} \]

\[ \tan(\theta_B) \]

\[ \tan(\theta_t) \]

\[ \tan(\theta_t) = (M+1) \tan(\theta_B) \]

Energy per row

\[ F(M) = \frac{1}{M+1} \left( (M+1)^3 \frac{K_J}{2} \tan^2 \theta_B + F_{M0} \right) \]

\[ M + 1 \approx \left( \frac{F_{M0}}{K_J \tan^2 \theta_B} \right)^{1/3} \]
Estimates

Number of straight rows $M$ in between tilted rows

$$M + 1 = \left( \frac{0.24 \Phi_0 \gamma^2 B_z}{4\pi\lambda^2 B_x^2} \right)^{1/3} \gg 1$$

Energy

$$F(B_x, B_z) - F(0, B_z) \approx \frac{\varepsilon_0}{2\pi\lambda^2} \left[ \frac{B_x}{\gamma B_z} \right]^{2/3} \left[ \frac{2\pi\lambda^2 B_z}{\Phi_0} \right]^{1/3}$$

Transition composite $\rightarrow$ tilted lattice $E_{M=1} = E_{\text{tilt}}$

$$B_{x,t}(B_z) \approx \gamma \sqrt{\frac{B_z \Phi_0}{4\pi\lambda^2}}$$
Phase diagram using analytical energies

\[ B_z (\text{in units } \Phi_0/4\pi\lambda^2) \]

\[ B_x (\text{in units } \Phi_0/2\pi\gamma s^2) \]

\[ \alpha = 0.8 \]

M=1

tilted

\[ 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \]

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Numerical exploration of lattice states

Numerical minimization of energy of unit cell with respect to
• row displacements $u_{n,l}$
• phase $\phi_n(y)$

$$E[\phi, u] = E_{\text{phase}}[\phi] + E_M[u] + E_{\text{shear}}[u] + E_J[\phi, u]$$

Cell $2N_l \times (2M_h + 1)$
Comparison of analytical and numerical results

\[ \alpha = 0.8, \ B=0.5\Phi_0/\gamma s^2 \]

\[ F \]

\[ B_x \text{ [in units } \Phi_0/(2\pi\gamma s^2)] \]

- numerical
- analytical

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Crossover from tilted to soliton rows

\[ \tan(\theta_t) \approx \gamma/10 \]

solitonlike

modulated

\[ \tan(\theta_0) \tan(\theta_t) \approx \gamma \]

\[ M = 6 \]

\[ M = 2 \]
BSCCO phase diagram

\[ H_{c1}[Oe] \]

\[ H_{ab}[Oe] \]

\[ \Delta B_m = \frac{E_M}{\Delta M} \approx 30 \text{ G} \]

magnetic-coupling shift

Parameters
\[ \lambda_{ab} = 300 \text{nm} \text{ and } \gamma = 200 \text{ (} \alpha = 0.96 \text{)} \]
give
\[ B_t \approx 1.7 \text{ kG} \text{ at } B_z = 80 \text{G} \]

role of fluctuations?

\[ \Delta B_m = \frac{E_M}{\Delta M} \approx 30 \text{ G} \]

Konczykowski et al., 2000
Mircovic et al., 2001
Tokunaga et al., 2002

small?

\[ \Delta B_m = \frac{E_M}{\Delta M} \approx 30 \text{ G} \]

magnetic-coupling shift

\[ E_M \approx -\frac{9\Phi_0^2}{256\pi^5\lambda^4} \exp \left( -\frac{8\pi^2c_L^2}{3} \right) \]
Summary

• Chain transitions
  – Transition crossing → tilted at moderate $B_z$ (1-3 G)
    • Intermediate states
  – Transition [kinked lines] → [crossing chain] at very small $B_z$ (0.1-0.3 G)
    • Finite anisotropy range $0.5 < \alpha < 0.65$
    • Density jump

• Crossover Josephson vortex-soliton at $\alpha \sim 0.5$

• Composite-lattice phase diagram for $\alpha \sim 1$
  – Solitons → Composite → Tilted