Coulomb effects in a mixed granular system near the percolation threshold

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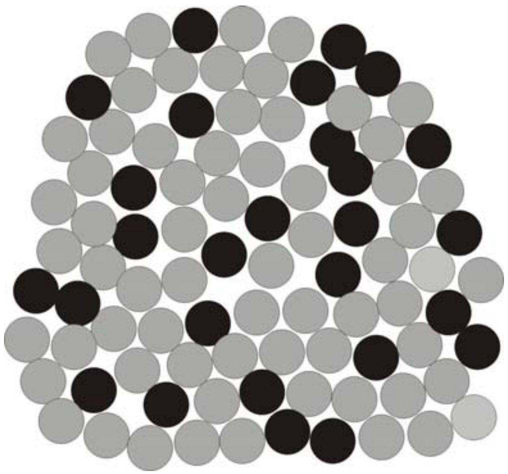
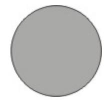
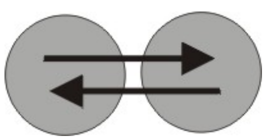
- The system: mixture of conducting and nonconducting grains.
- Coulomb blocade and special role of critical clusters in transport near the percolation threshold.
- Arrhenius, Mott, Efros-Shklovskii, and more exotic laws.
- Coulomb anomaly in cluster-cluster transitions: specifics of fractality and possibility for nontrivial stretched exponential T-dependence of conductivity.

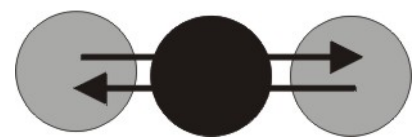
Mixture of conducting and nonconducting grains



 -- metal grain.  -- nonmetal grain.

$$E_C \sim e^2/2C$$ -- charging energy.





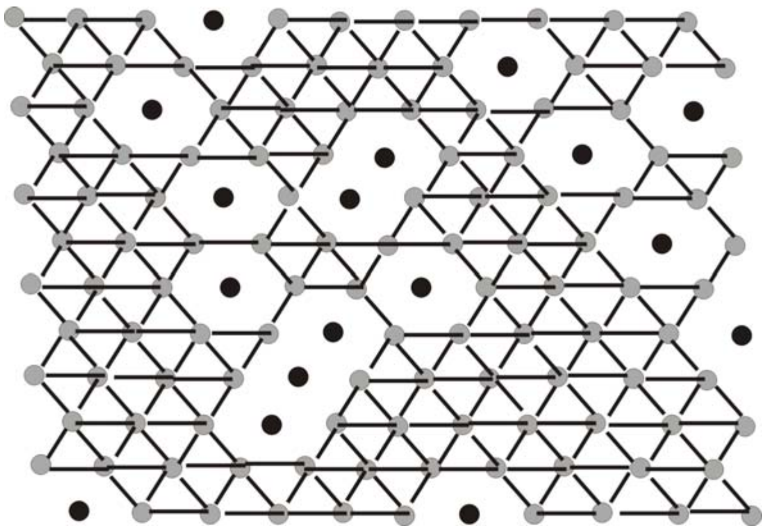
Tunneling via intermediate nonmetal grain: Conductance

$$g \ll G$$

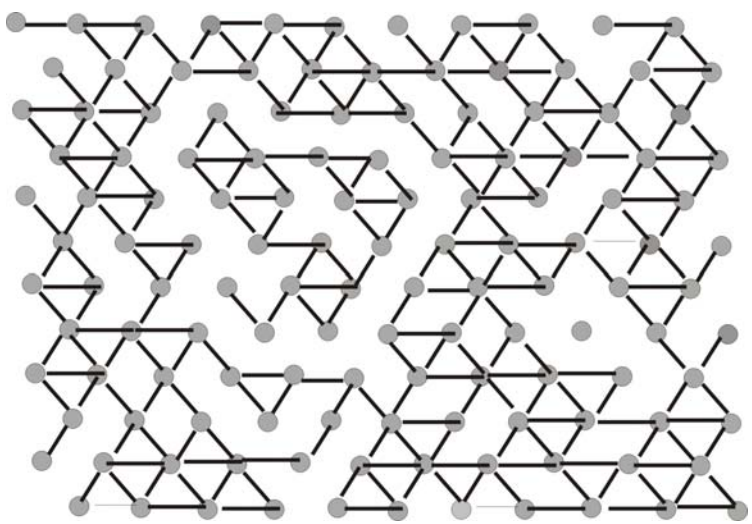
Direct tunneling between metal grains: conductance G

We will consider the case $G \gg 1, \quad g \ll 1$

Standard percolation approach, no Coulomb effects: $T \gg E_C$



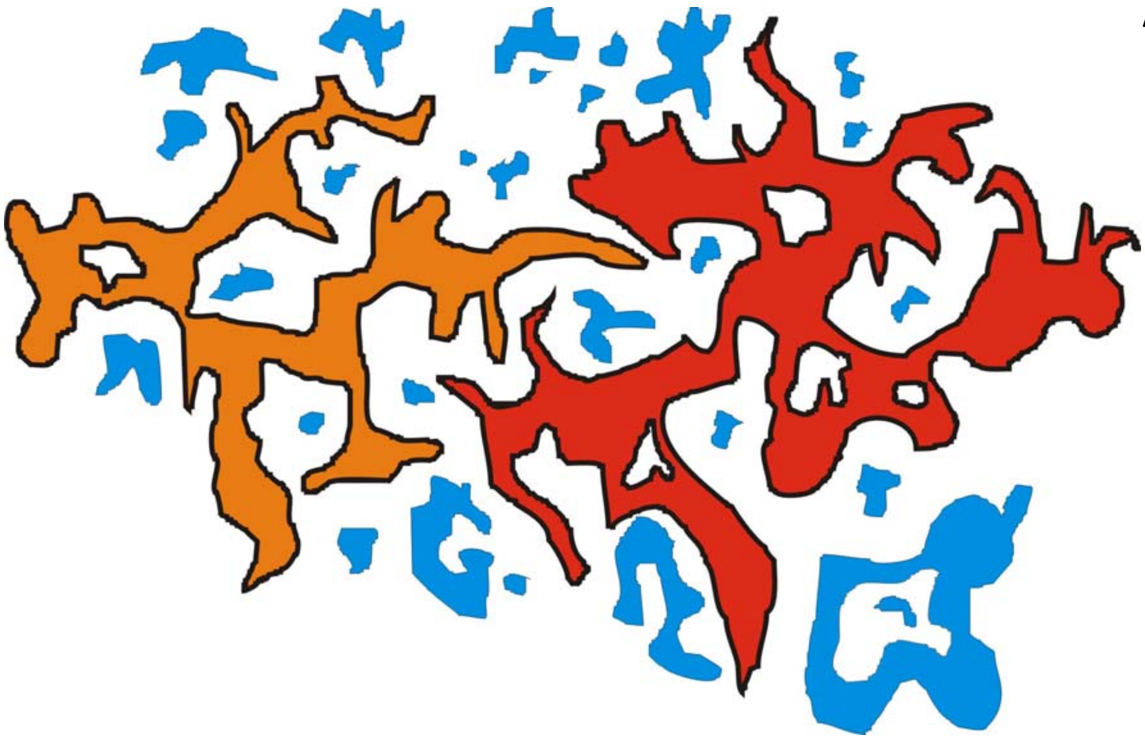
“Site percolation” model:
randomly chosen sites are disconnected.

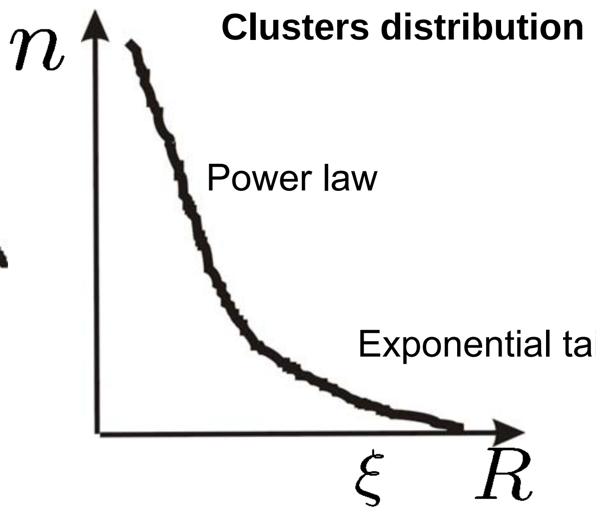


“Bond percolation” model:
randomly chosen bonds are destroyed.

For simplicity: $g \longrightarrow 0$

Structure of conducting clusters just below the percolation threshold



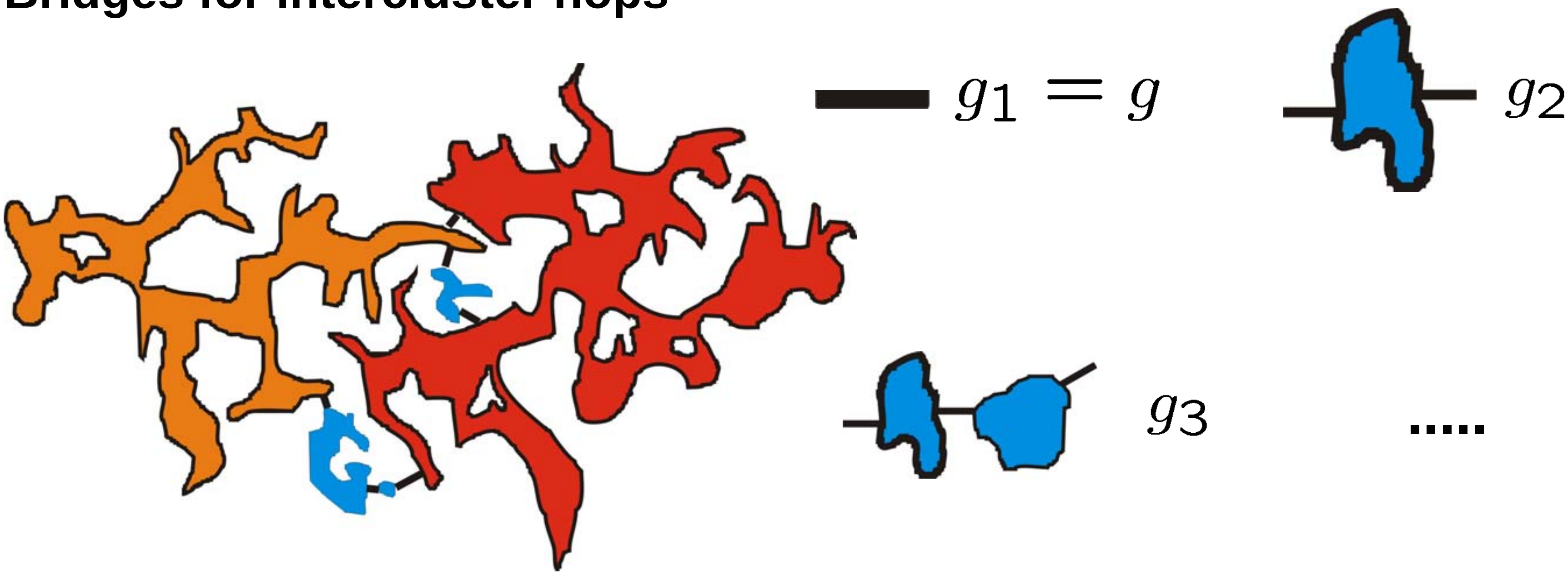


- Most sites belong to small clusters.
- Extremely large clusters with size $> \xi$ are rare.
- Conducting electrons stay predominantly on critical clusters with size $\sim \xi$.

Intercluster hops necessarily involve poor conductances g .

Here the Coulomb interaction comes into play!

Bridges for intercluster hops



No Coulomb $(T \gg E_C)$

$$g_n \approx \frac{g}{n}$$

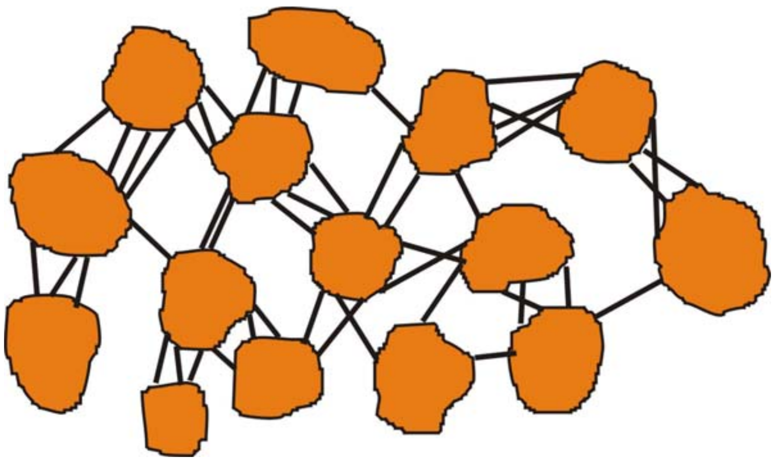
Considerable contribution of multilink hops, small clusters participate in transport.

Strong Coulomb $(T \ll E_C)$

$$g_n \sim g \,\mathrm{max} \left\{ e^{-\frac{E_C}{T}}, g\left(\frac{T}{E_C}\right)^2 \right\}^{(n-1)}$$

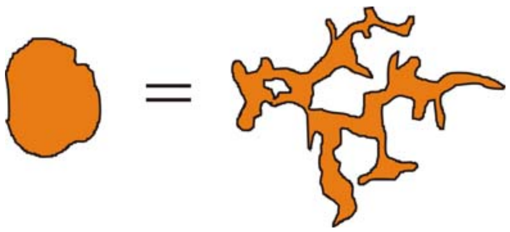
Mutilink hops are suppressed by Coulomb blocade or inelastic cotunneling.
Small clusters are irrelevant.

Effective network of critical clusters



$$N_{\text{link}} \sim (\xi/a)^{d_{\text{link}}} \gg 1$$

-- average number of single-link bridges between two neighboring critical clusters.



(critical cluster)

$G^*(\xi) \sim G(a/\xi)^{\tilde{\zeta}} \ll G$ -- classic conductance across a critical cluster.

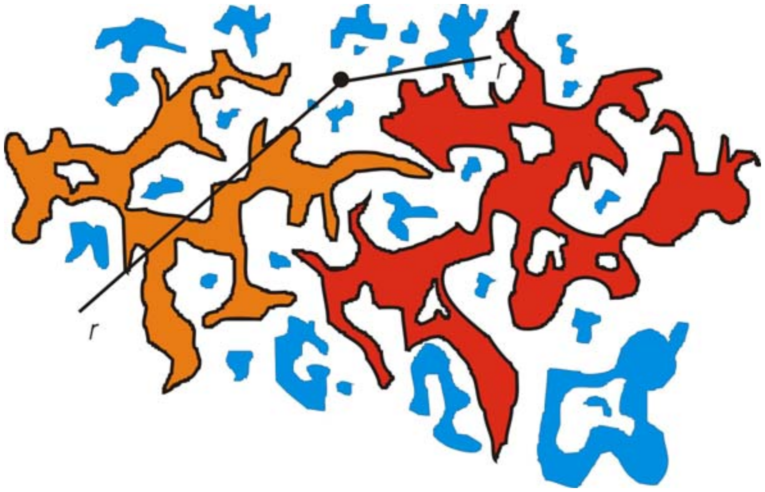
$g^*(\xi) \sim N_{\text{link}} \gg g$ -- effective intercluster conductance.

We will assume that still $g^*(\xi) \ll 1,\ \ G^*(\xi)$, but $G^*(\xi)$ can be arbitrary.

- For $G^*(\xi) \gg 1$ clusters are effectively point-like objects.

- For $G^*(\xi) \ll 1$ clusters are extended objects with internal degrees of freedom.

Dielectric screening by polarized clusters



The only role of subcritical clusters – screening of Coulomb interaction

$$\tilde{s} = \begin{cases} 0.974, & \text{for } d = 2 \\ 0.85, & \text{for } d = 3 \end{cases}$$

Fractal range $a \ll r \ll \xi$

$$U(r) \sim \frac{e}{\epsilon r}\left(\frac{a}{r}\right)^{\tilde{s}}$$

Main contribution:
subcritical clusters of size r

Self-averaged range $r \gg \xi$

$$U(r) \sim \frac{e}{\epsilon r}\left(\frac{a}{\xi}\right)^{\tilde{s}}$$

Main contribution:
critical clusters of size ξ

Charging energy for critical clusters: $E_C^*(\xi) \sim E_C(a/\xi)^{\tilde{s}+1} \ll E_C$

Large effective conductance $G^*(\xi) \gg 1$

Mapping onto the standard granular system is possible with $g \to g^*(\xi), \quad E_C \to E_C^*(\xi)$

Two characteristic temperatures: $T_0 \sim E_C^*(\xi), \quad T_{ES} \sim E_C^*(\xi)/\mathcal{L}$

where $\mathcal{L} = \ln \left[\left(E_C^*(\xi)/T \right)^2 / g^*(\xi) \right]$ -- large logarithm.

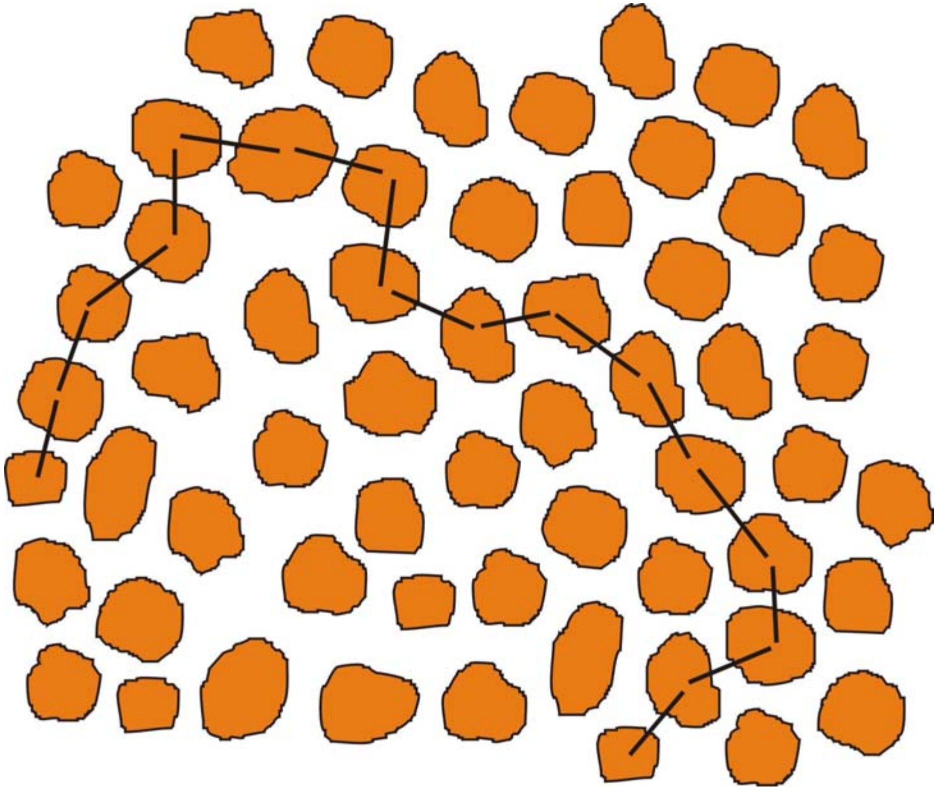
- $T_{ES} < T < T_0$ -- nearest neighbor hopping via critical clusters

$$\sigma \sim \xi^{2-d} g^*(\xi) \exp \left\{ -E_C^*(\xi)/T \right\}$$

- $T < T_{ES}$ -- variable range inelastic cotunneling via resonant clusters

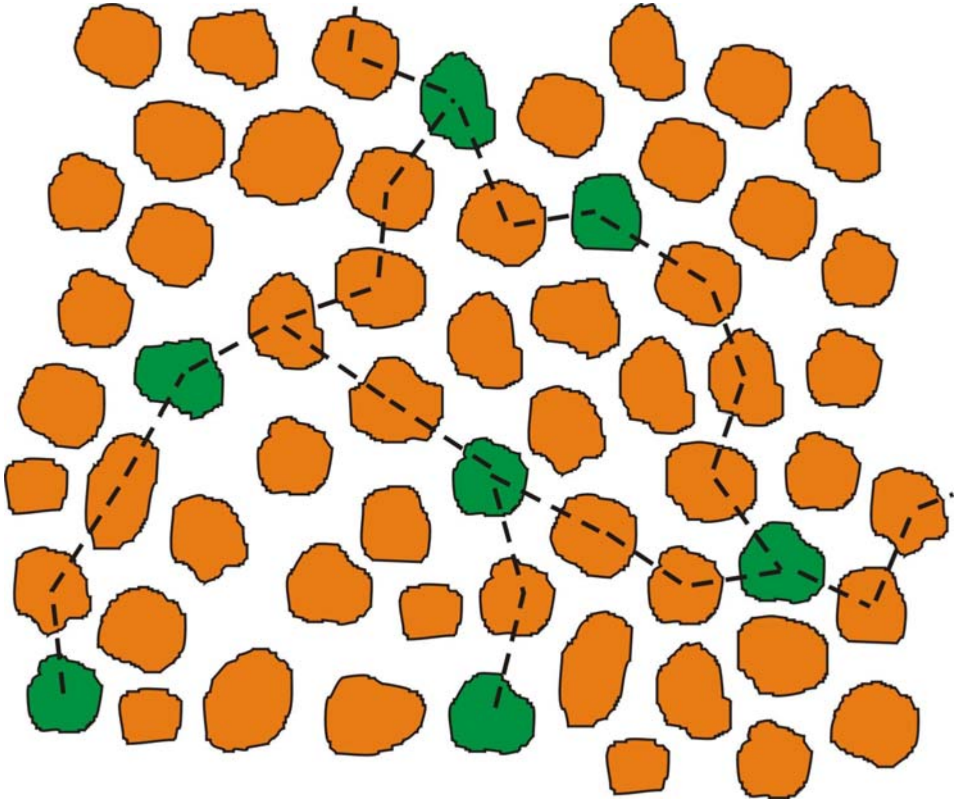
$$\sigma \sim \xi^{2-d} g^*(\xi) \exp \left\{ -(T_{ES}/T)^{1/2} \right\}$$

Nearest neighbor hopping



Each hop is a real thermoactivated intergrain transition.

Variable range cotunneling



Real states only on resonant (green) grains with anomalously small E_C^*

Intermediate hops (dashed lines) involve virtual states.

Small effective conductance $G^*(\xi) \ll 1.$

Three characteristic temperatures:

$$T_0 \sim E_C G^{-\gamma} \equiv E_C^*(\xi) G^*(\xi)^{-\gamma}, \quad \gamma = \begin{cases} 2.00, & \text{for } d=2 \\ 1.44, & \text{for } d=3 \end{cases}$$

$$T_c \sim E_C^*(\xi) G^*(\xi) \ll T_0, \qquad T_{ES} \sim T_c/\mathcal{L} \ll T_c.$$

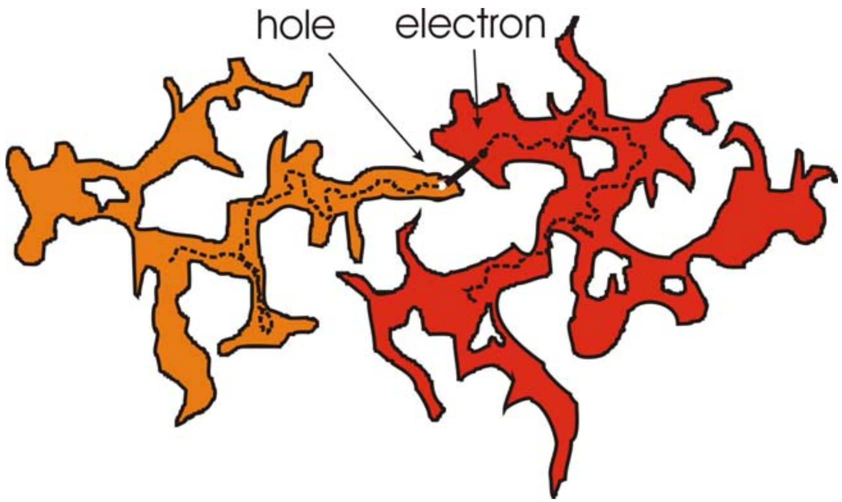
- $T_c < T < T_0$ -- Coulomb zero-bias-amomaly mechanism for intercluster hops:

$$\sigma \sim \xi^{2-d} g^*(\xi) \exp\left\{-(T_0/T)^{\tilde{\alpha}}\right\}, \quad \tilde{\alpha} = \begin{cases} 0.33, & \text{for } d=2 \\ 0.41, & \text{for } d=3 \end{cases}$$

- $T_{ES} < T < T_c$ -- nearest neighbor hopping via critical clusters (the same as before, with the same activation energy)

- $T < T_{ES}$ -- variable range inelastic cotunneling via extended objects (the same formula, as before, but *with new* T_{ES})

The Coulomb anomaly regime



Electron and hole have yet to tunnel from under the Coulomb barrier.

This is a diffusion on a fractal !

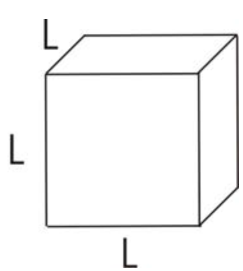
An approach a la Levitov and Shytov leads to suppression of intercluster hopping rate:

$$g^*(\xi) \to g^*(\xi)e^{-S}$$

$$S \sim \int_T \frac{d\omega}{2\pi\omega} \int_{1/\xi} \frac{d^d\mathbf{q}}{(2\pi)^d} \frac{U_q}{\omega + \sigma_q q^2 U_q}, \quad \sigma_q \sim \sigma^{(0)}(qa)^{\tilde{\mu}}, \quad U_q \sim \frac{(qa)^{\tilde{s}}}{\epsilon d^{d-1}}$$

Manifestations of fractality: $\tilde{\mu} \neq 0, \quad \tilde{s} \neq 0.$

Definitions: resistance of d-dimensional fractal cube:



$$R(L) \sim L^{\tilde{\zeta}}, \quad \tilde{\zeta} = \begin{cases} 0.974, & \text{for } d = 2 \\ 1.3, & \text{for } d = 3 \end{cases}$$

$$\sigma(L) \sim L^{2-d}/R(L) \sim L^{-\tilde{\mu}}, \qquad \tilde{\zeta} = \tilde{\mu} + 2 - d.$$

Peculiarities of the fractal case

$$S \sim \int_T \frac{d\omega}{2\pi\omega} \int_{1/\xi} \frac{d^d\mathbf{q}}{(2\pi)^d} \frac{U_q}{\omega + \sigma_q q^2 U_q}, \qquad \sigma_q \sim \sigma^{(0)}(qa)^{\tilde{\mu}}, \;\; U_q \sim \frac{(qa)^{\tilde{s}}}{\epsilon d^{d-1}}.$$

- The integral has ifrared power-law divergence for any dimension $d = 1,\, 2,\, 3.$
  The reason – slowing down of diffusion on fractal at $t \to \infty$ or $r \to \infty$.

- For $T > T_c$ the integral converges at $\omega \sim T, \;\; q \sim (T/GE_C)^{1/(\tilde{\zeta}+\tilde{s}+1)} \gg \xi^{-1}.$
  Thus, the action does not depend on ξ. The tunneling stops before the charge has time to spread over the entire cluster.

- As a result, $S \sim \left(\frac{T_0}{T}\right)^{\tilde{\alpha}}, \;\; \tilde{\alpha} = \frac{\tilde{\zeta}}{\tilde{\zeta}+\tilde{s}+1}.$

Fractal case:

$$\tilde{\zeta} > 0$$

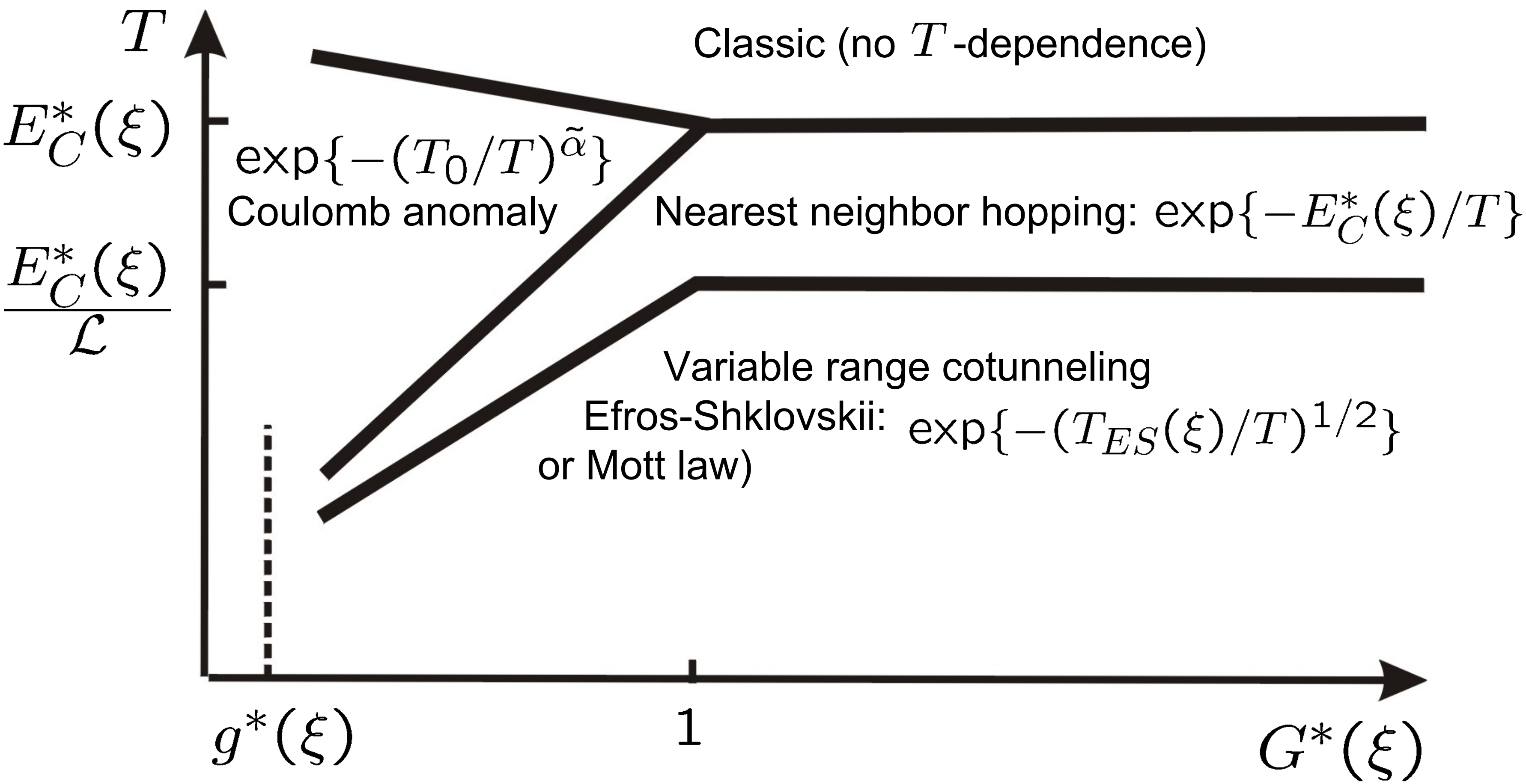
in all dimensions.

Nonfractal case:

$$\tilde{\zeta} = \begin{cases} 1, & \text{for } d = 1, \text{ infrared singularity} \\ 0, & \text{for } d = 2, \text{ logarithmic case} \\ -1, & \text{for } d = 3, \text{ ultraviolet singularity} \end{cases}$$

- For $T < T_c$ the spatial integral converges at $q \sim \xi^{-1}$, here $S \sim E_C^*/T.$

The phase diagram



- All lines are crossovers.
- The Mott law can only be observed for low level of charge disorder.
- To the left from the dashed line – critical regime. T - dependence --???

Conclusions

- The conduction process near percolation threshold involves direct hops of electrons between large critical clusters. Small clusters are excluded from this process due to Coulomb blocade.

- If the conductance across a critical cluster $G^*(\xi) \gg 1$, then the system is equivalent to “simple” granular metal with critical clusters as “supergrains”.

- For $G^*(\xi) \ll 1$ an unusual stretch-exponential law for conductivity

$$\sigma \propto \exp\left\{-(T_0/T)^{\tilde{\alpha}}\right\}, \quad \tilde{\alpha} = \begin{cases} 0.33, & \text{for } d = 2 \\ 0.41, & \text{for } d = 3 \end{cases}$$

may be observed due to Coulomb zero-bias anomaly effect, dramatically enhanced by fractality of critical clusters.