Coulomb effects in a mixed granular system near the percolation threshold

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- The system: mixture of conducting and nonconducting grains.
- Coulomb blockade and special role of critical clusters in transport near the percolation threshold.
- Arrhenius, Mott, Efros-Shklovskii, and more exotic laws.
- Coulomb anomaly in cluster-cluster transitions: specifics of fractality and possibility for nontrivial stretched exponential $T$-dependence of conductivity.
Mixture of conducting and nonconducting grains

-- metal grain.

-- nonmetal grain.

\[ E_C \sim \frac{e^2}{2C} \]

-- charging energy.

Direct tunneling between metal grains: conductance \( G \)

Tunneling via intermediate nonmetal grain: Conductance \( g \ll G \)

We will consider the case \( G \gg 1, \ g \ll 1 \)
Standard percolation approach, no Coulomb effects: \( T \gg E_C \)

“Site percolation” model:
randomly chosen sites are disconnected.

“Bond percolation” model:
randomly chosen bonds are destroyed.

For simplicity: \( g \rightarrow 0 \)
Structure of conducting clusters just below the percolation threshold

- Most sites belong to small clusters.
- Extremely large clusters with size $\xi$ are rare.
- Conducting electrons stay predominantly on critical clusters with size $\sim \xi$.

Intercluster hops necessarily involve poor conductances $g$.

Here the Coulomb interaction comes into play!
Bridges for intercluster hops

\[ g_n \approx \frac{g}{n} \]

Considerable contribution of multilink hops, small clusters participate in transport.

\[ g_n \sim g \max \left\{ e^{-\frac{E_C}{T}}, g \left( \frac{T}{E_C} \right)^2 \right\}^{n-1} \]

Mutilink hops are suppressed by Coulomb blockade or inelastic cotunneling. Small clusters are irrelevant.
Effective network of critical clusters

\[ N_{\text{link}} \sim (\xi/a)^{d_{\text{link}}} \gg 1 \]

-- average number of single-link bridges between two neighboring critical clusters.

\[ G^*(\xi) \sim G(a/\xi)^{\tilde{\zeta}} \ll G \]

-- classic conductance across a critical cluster.

\[ g^*(\xi) \sim N_{\text{link}} \gg g \]

-- effective intercluster conductance.

We will assume that still \( g^*(\xi) \ll 1, \quad G^*(\xi) \), but \( G^*(\xi) \) can be arbitrary.

- For \( G^*(\xi) \gg 1 \) clusters are effectively point-like objects.

- For \( G^*(\xi) \ll 1 \) clusters are extended objects with internal degrees of freedom.
Dielectric screening by polarized clusters

The only role of subcritical clusters – screening of Coulomb interaction

Fractal range \( a \ll r \ll \xi \)

\[
U(r) \sim \frac{e}{\varepsilon r} \left( \frac{a}{r} \right)^{\tilde{s}}
\]

Main contribution: subcritical clusters of size \( r \)

Self-averaged range \( r \gg \xi \)

\[
U(r) \sim \frac{e}{\varepsilon r} \left( \frac{a}{\xi} \right)^{\tilde{s}}
\]

Main contribution: critical clusters of size \( \xi \)

Charging energy for critical clusters:

\[
E_C^*(\xi) \sim E_C(a/\xi)^{\tilde{s}+1} \ll E_C
\]
Large effective conductance \( G^*(\xi) \gg 1 \)

Mapping onto the standard granular system is possible with \( g \to g^*(\xi), \quad E_C \to E^*_C(\xi) \)

Two characteristic temperatures:
\[
T_0 \sim E^*_C(\xi), \quad T_{ES} \sim E^*_C(\xi)/\mathcal{L}
\]

where \( \mathcal{L} = \ln \left[ \left( \frac{E^*_C(\xi)}{T} \right)^2 / g^*(\xi) \right] \) -- large logarithm.

- \( T_{ES} < T < T_0 \) -- nearest neighbor hopping via critical clusters
  \[
  \sigma \sim \xi^{2-d} g^*(\xi) \exp \left\{ -\frac{E^*_C(\xi)}{T} \right\}
  \]

- \( T < T_{ES} \) -- variable range inelastic cotunneling via resonant clusters
  \[
  \sigma \sim \xi^{2-d} g^*(\xi) \exp \left\{ -(T_{ES}/T)^{1/2} \right\}
  \]
Nearest neighbor hopping

Each hop is a real thermoactivated intergrain transition.
Variable range cotunneling

Real states only on resonant (green) grains with anomalously small $E^*_C$

Intermediate hops (dashed lines) involve virtual states.
Small effective conductance $G^*(\xi) \ll 1$.

Three characteristic temperatures:

$$T_0 \sim E_C G^{-\gamma} \equiv E_C^*(\xi) G^*(\xi)^{-\gamma}, \quad \gamma = \begin{cases} 2.00, & \text{for } d = 2 \\ 1.44, & \text{for } d = 3 \end{cases}$$

$$T_c \sim E_C^*(\xi) G^*(\xi) \ll T_0, \quad T_{ES} \sim T_c/L \ll T_c.$$

- $T_c < T < T_0$ -- Coulomb zero-bias-anomaly mechanism for intercluster hops:
  $$\sigma \sim \xi^{2-d} g^*(\xi) \exp \left\{ -(T_0/T)\bar{\alpha} \right\}, \quad \bar{\alpha} = \begin{cases} 0.33, & \text{for } d = 2 \\ 0.41, & \text{for } d = 3 \end{cases}$$

- $T_{ES} < T < T_c$ -- nearest neighbor hopping via critical clusters
  (the same as before, with the same activation energy)

- $T < T_{ES}$ -- variable range inelastic cotunneling via extended objects
  (the same formula, as before, but with new $T_{ES}$)
The Coulomb anomaly regime

Electron and hole have yet to tunnel from under the Coulomb barrier.

This is a diffusion on a fractal!

An approach a la Levitov and Shytov leads to suppression of intercluster hopping rate:

$$g^*(\xi) \to g^*(\xi) e^{-S}$$

$$S \sim \int_T \frac{d\omega}{2\pi\omega} \int_{1/\xi} \frac{d^d q}{(2\pi)^d \omega} \frac{U_q}{\sigma_q q^2 U_q}, \quad \sigma_q \sim \sigma^{(0)}(qa)^{\tilde{\mu}}, \quad U_q \sim \frac{(qa)^{\tilde{s}}}{\epsilon a^{d-1}}$$

Manifestations of fractality: $\tilde{\mu} \neq 0$, $\tilde{s} \neq 0$.

Definitions: resistance of d-dimensional fractal cube:

$$R(L) \sim L^{\tilde{\zeta}}, \quad \tilde{\zeta} = \begin{cases} 0.974, & \text{for } d = 2 \\ 1.3, & \text{for } d = 3 \end{cases}$$

$$\sigma(L) \sim L^{2-d}/R(L) \sim L^{-\tilde{\mu}}, \quad \tilde{\zeta} = \tilde{\mu} + 2 - d.$$
Peculiarities of the fractal case

\[ S \sim \int_T \frac{d\omega}{2\pi\omega} \int \frac{d^d q}{(2\pi)^d} \frac{U_q}{\sigma_q + \sigma_q^2 U_q}, \quad \sigma_q \sim \sigma^{(0)}(q\tilde{a})^{\tilde{\mu}}, \quad U_q \sim \frac{(q\tilde{a})^{\tilde{s}}}{e^{d-1}}. \]

- The integral has infrared power-law divergence for any dimension \( d = 1, 2, 3 \). The reason – slowing down of diffusion on fractal at \( t \to \infty \) or \( r \to \infty \).

- For \( T > T_c \) the integral converges at \( \omega \sim T, \quad q \sim (T/GE_C)^{1/(\tilde{\zeta}+\tilde{s}+1)} \gg \xi^{-1} \).

  Thus, the action does not depend on \( \xi \). The tunneling stops before the charge has time to spread over the entire cluster.

- As a result, \( S \sim \left( \frac{T_0}{T} \right)^{\tilde{\alpha}}, \quad \tilde{\alpha} = \frac{\tilde{\zeta}}{\tilde{\zeta}+\tilde{s}+1} \).

Fractal case: \( \tilde{\zeta} > 0 \)

in all dimensions.

Nonfractal case: \( \tilde{\zeta} = \begin{cases} 1, & \text{for } d = 1, \text{ infrared singularity} \\ 0, & \text{for } d = 2, \text{ logarithmic case} \\ -1, & \text{for } d = 3, \text{ ultraviolet singularity} \end{cases} \)

- For \( T < T_c \) the spatial integral converges at \( q \sim \xi^{-1} \), here \( S \sim E_C^*/T \).
The phase diagram

- All lines are crossovers.
- The Mott law can only be observed for low level of charge disorder.
- To the left from the dashed line – critical regime. $T$ - dependence --???
Conclusions

- The conduction process near percolation threshold involves direct hops of electrons between large critical clusters. Small clusters are excluded from this process due to Coulomb blockade.

- If the conductance across a critical cluster $G^*(\xi) \gg 1$, then the system is equivalent to “simple” granular metal with critical clusters as “supergrains”.

- For $G^*(\xi) \ll 1$ an unusual stretch-exponential law for conductivity

$$\sigma \propto \exp \left\{ -\left(\frac{T_0}{T}\right)^{\tilde{\alpha}} \right\}, \quad \tilde{\alpha} = \begin{cases} 0.33, & \text{for } d = 2 \\ 0.41, & \text{for } d = 3 \end{cases}$$

may be observed due to Coulomb zero-bias anomaly effect, dramatically enhanced by fractality of critical clusters.