

V.F. Gantmakher and V.T. Dolgoplov

**Quantum phase transitions of
'localized – delocalized electrons'
type**

1. Metal-insulator transitions

3D-system of non-interacting electrons

2D-system of non-interacting electrons

2D-system of electrons with spin-orbit interaction

2D-system of interacting electrons

2. Superconductor-insulator transition

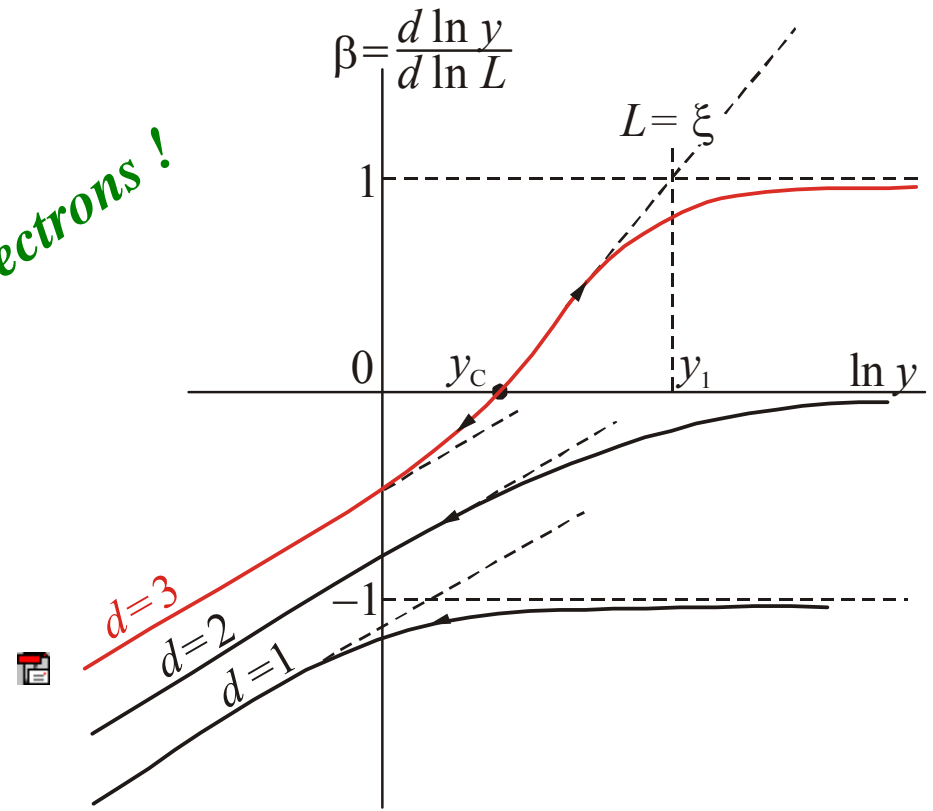
*E. Abrahams, P.W. Anderson,
D.C. Licciardello, and
T.W. Ramakrishnan,
Phys.Rev.Lett. 42, 673 (1979)*

(AALR-1979)

Non-interacting electrons !

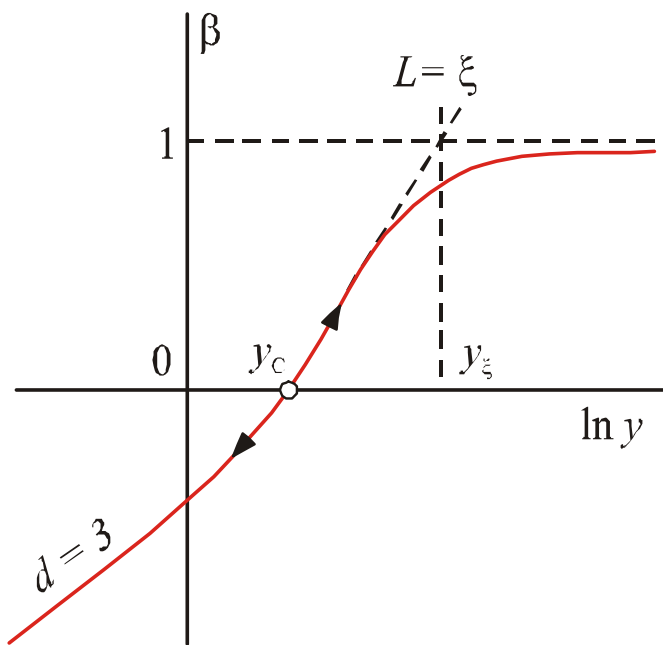
With $d=3$,

1. at y_c the metal-insulator transition takes place;
2. the value of the three-dimensional conductivity of a rather large sample may be infinitesimal;
3. the metal--insulator transition is continuous.

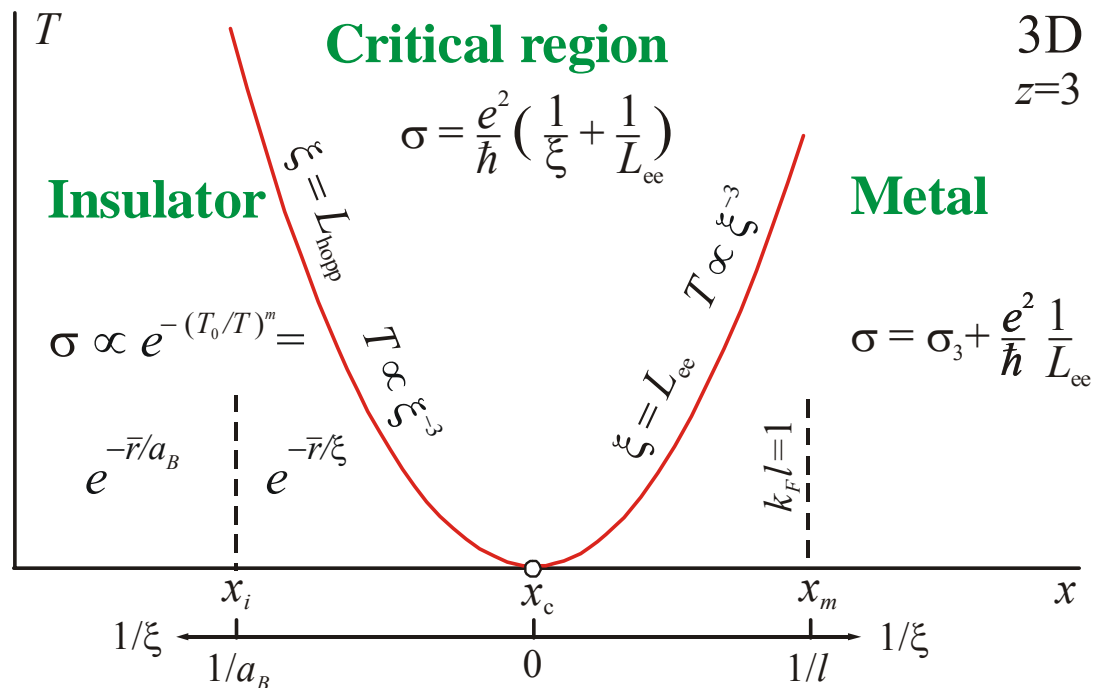


In essence, the flow diagram plays role of existence theorem with respect to metal-insulator transition

3D-system of non-interacting electrons



Flow diagram



(x, T) - diagram

$d = 3$

The function $\sigma(T)$ in the critical region

In the metallic region at $T \neq 0$

$$\sigma = \sigma_{03} + \frac{e^2}{\hbar} \frac{1}{L_T} \quad L_T = \sqrt{D\hbar/T},$$

Near the transition at $T = 0$

$$\sigma = \frac{e^2}{\hbar} \frac{1}{\xi}$$

Interpolation formula

$$\sigma = \frac{e^2}{\hbar} \left(\frac{1}{\xi} + \frac{1}{L_T} \right)$$

$$\left\{ \begin{array}{l} \sigma = \frac{e^2}{\hbar} \left(\frac{1}{\xi} + \frac{1}{L_T} \right) \\ \sigma = e^2 g_F D \end{array} \right.$$

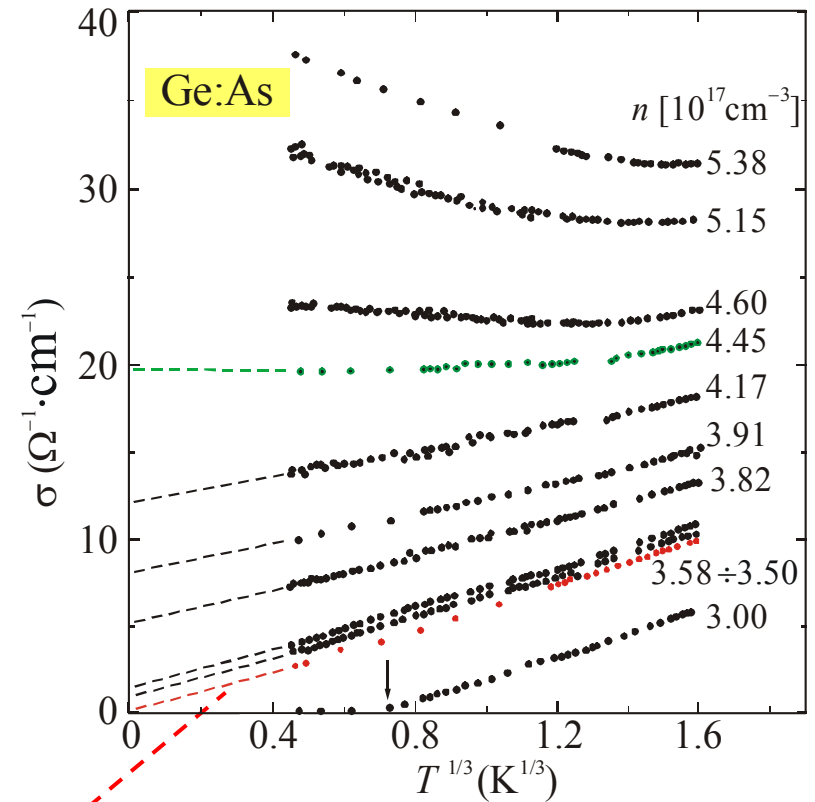
$$\xrightarrow{\xi \rightarrow \infty}$$

$$\sigma = \frac{e^2}{\hbar} (Tg_F)^{1/3}$$

Einstein relation

*I.Shlimak, M.Kaveh, R.Ussyshkin, et al., Phys.Rev.Lett. 77, 1103 (1996);
J.Phys.:Cond.Matt. 9, 9873 (1997)*

*This is not the critical
concentration*

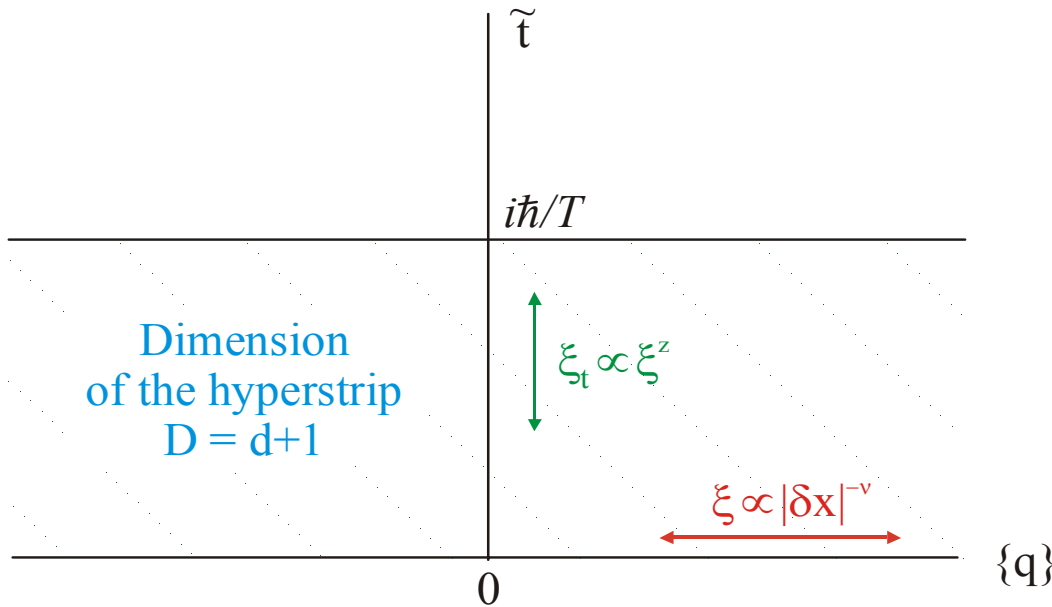


At the transition,

$$\frac{\partial \sigma}{\partial T} \neq 0$$

The mapping

of the quantum phase transition in d -space
to the classical phase transition in
 $D=d+1$ -dimensional strip



The partition function

$$Z = \sum_i e^{-\varepsilon_i/T} = \sum_i \int \varphi_i^*(q) \varphi_i(q) e^{-\varepsilon_i/T} dq$$

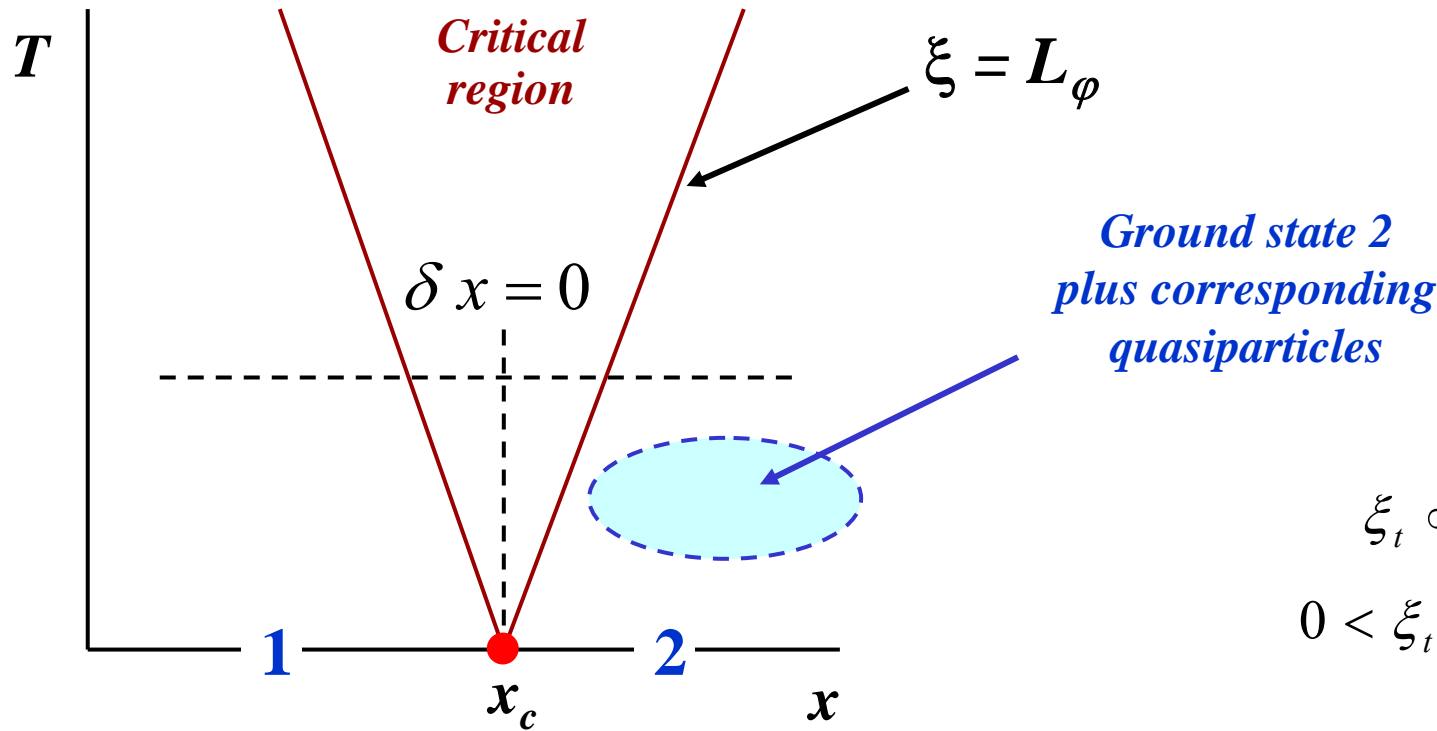
Diagonal matrix elements of time-evolution operator

$$\int \varphi_i^*(q) \varphi_i(q) e^{-i\varepsilon_i t/\hbar} dq$$

Two correlation lengths: ξ and ξ_t .

Hence, two scales: ξ and $L_\phi \propto \xi_t^{1/z} \propto (\hbar/T)^{1/z}$

Vicinity of the quantum phase transition



$$\left. \begin{aligned} \xi_t &\propto \xi^{1/z} \\ 0 < \xi_t < \hbar/T \end{aligned} \right\}$$

In the simplest case

$$\xi \propto (\delta x)^{-\nu}$$

Two lengths: ξ and $L_\phi \propto \xi^{1/z}$

Conductivity

$$\sigma \propto \xi^{2-d} f\left(\frac{L_\phi}{\xi}\right) \propto \xi^{2-d} f_1\left(\frac{\delta x}{T^{1/z\nu}}\right)$$

3D – metal - insulator transition

AALR

QPT

Theoretical scheme

* *experimental data*

Driving factor

Disorder

Interaction

Guideline

Transport

A thermodynamical function

Features in common

* *Two lengths, ξ and L_φ , in the vicinity of the transition*
Similar shape of the critical region

Scaling variable

*

$$\sigma = \frac{e^2}{\hbar} \left(\frac{1}{\xi} + \frac{1}{L_\varphi} \right)$$

$$\sigma \propto \xi^{-1} f\left(\frac{L_\varphi}{\xi}\right)$$

Graphical expression

Flow diagram

(x, T) – diagram

x – control parameter

Area of application

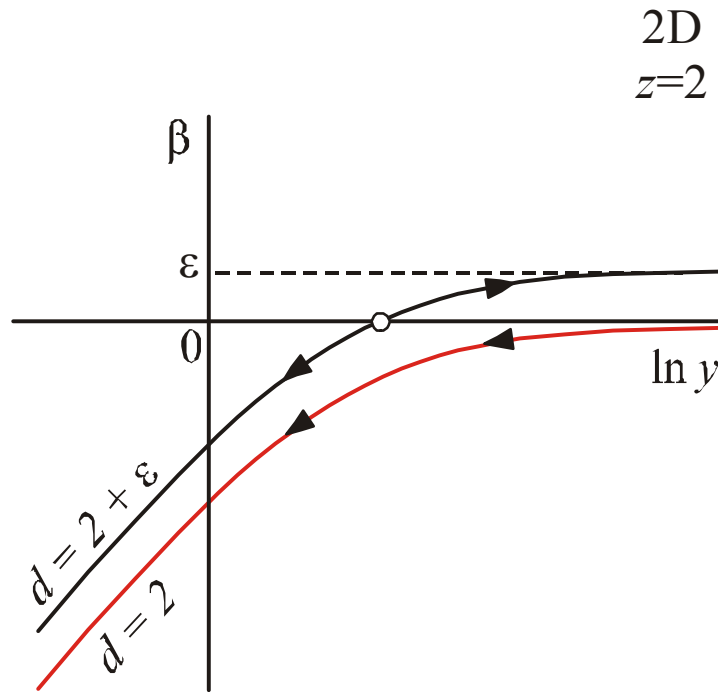
Global

Vicinity of the transition

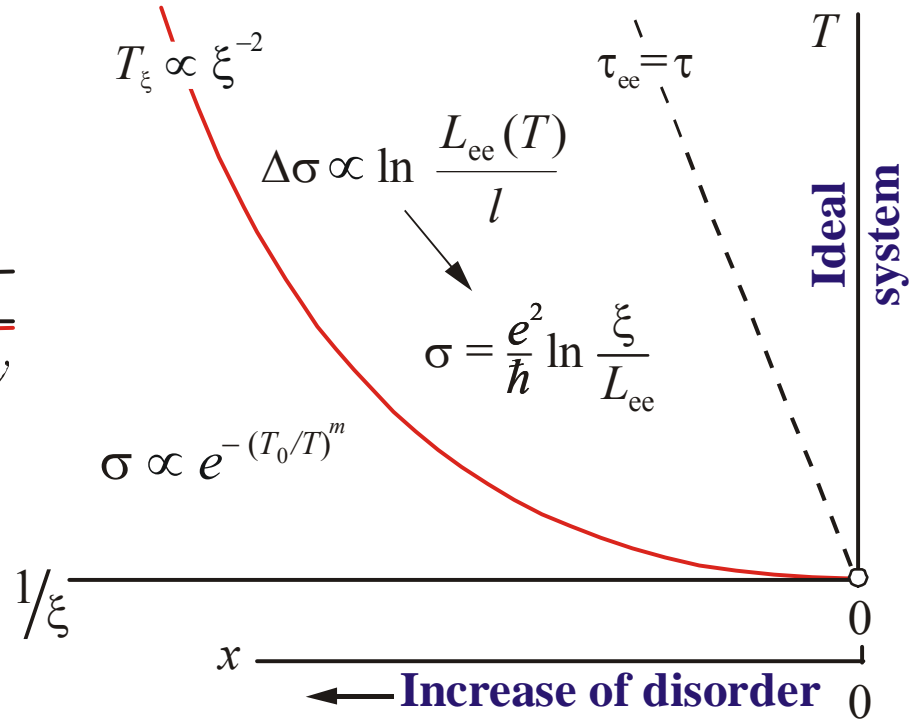
2D-system of non-interacting electrons

$$\left. \begin{aligned}
 \sigma &= \sigma_{02} - \frac{e^2}{\hbar} \ln \frac{L_T}{l} \\
 n &\propto k_F^2 \quad \sigma_{02} = \frac{ne^2 l}{\hbar k_F} \approx \frac{e^2}{\hbar} (k_F l)
 \end{aligned} \right\} \sigma = 0$$

$$k_F l = \ln \frac{L_T}{l} \quad L_T \equiv \xi = l \exp(k_F l)$$

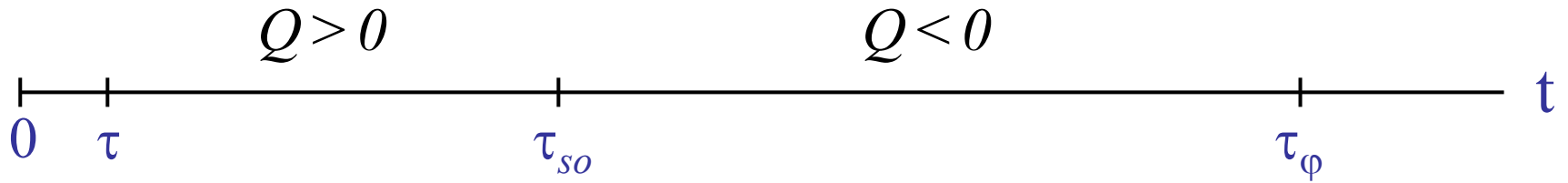


Flow diagram



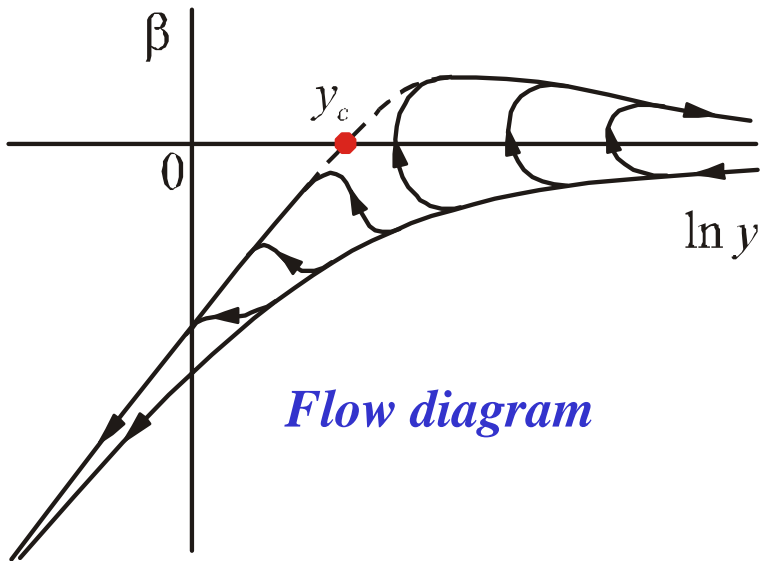
(x, T) - diagram

2D-system of electrons with spin-orbit interaction



$$\frac{\Delta\sigma}{\sigma} \approx - \int_{\tau}^{\tau_{\phi}} \frac{v_F \lambda^2 dt}{(Dt)b} \left(\frac{3}{2} e^{-t/\tau_{so}} - \frac{1}{2} \right)$$

Q



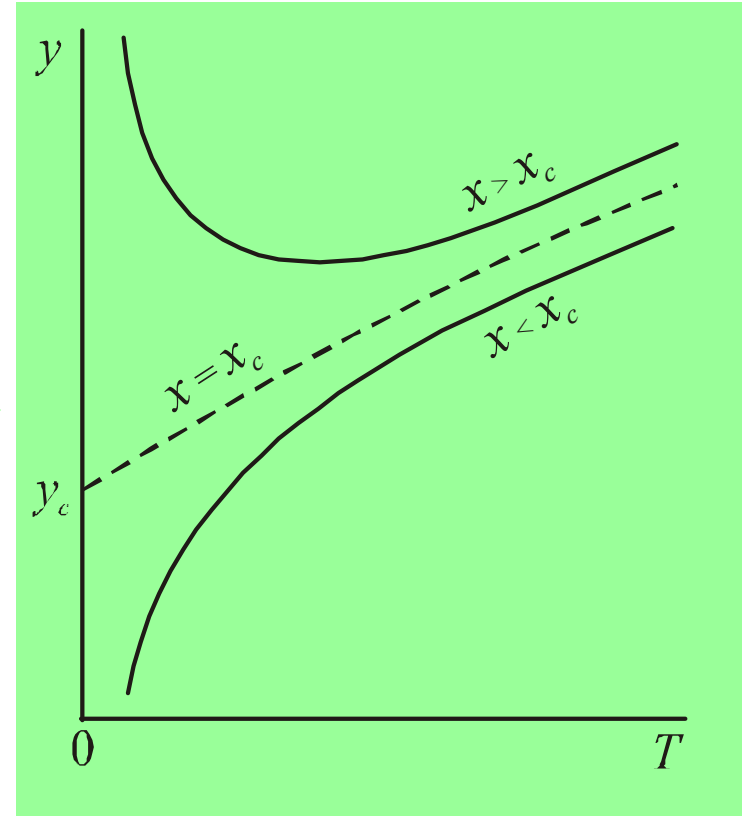
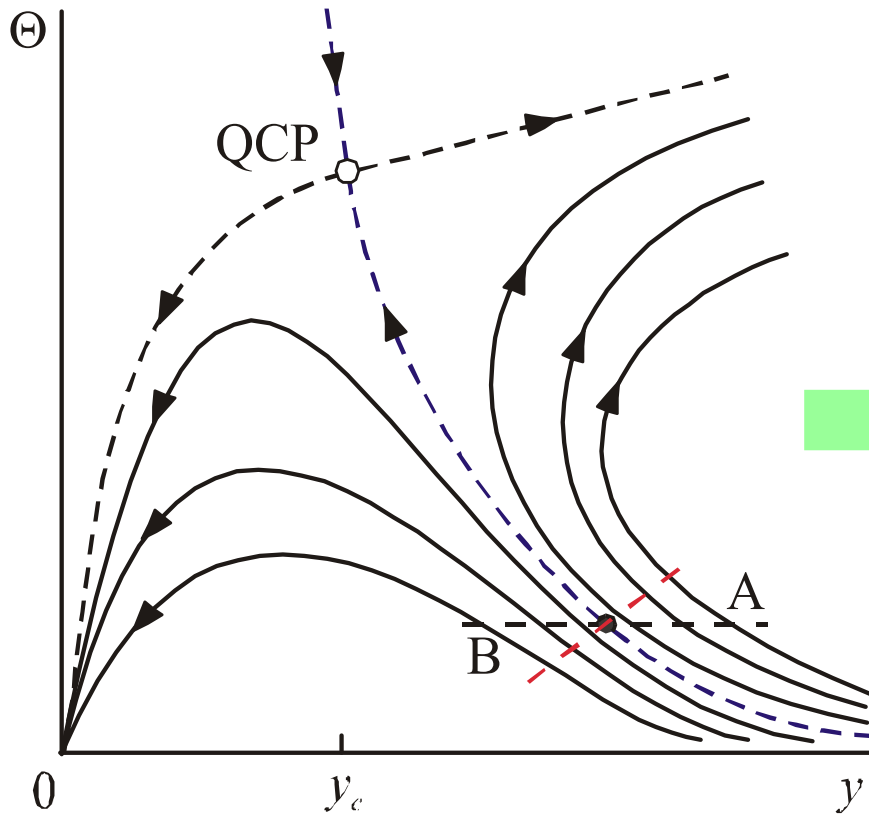
Flow diagram

Estimate for the transition point y_c

$$k_F l = \ln \tau^* / \tau \quad \tau^* = \tau \exp(k_F l)$$

If $\tau \ll \tau_{so} \ll \tau^* = \tau e^{k_F l}$, the film remains metallic

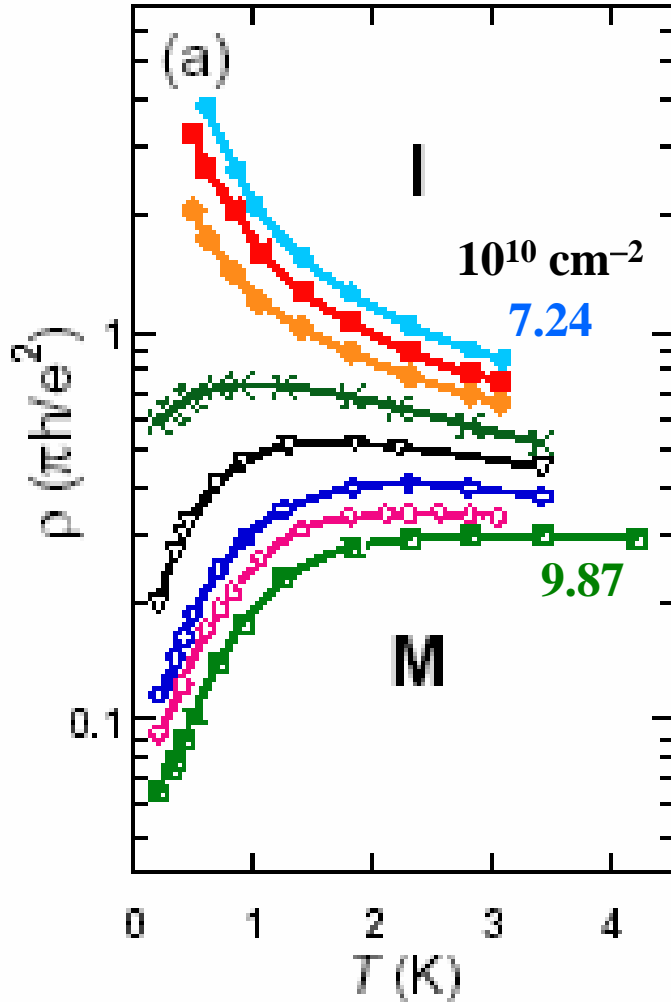
2D-system of interacting electrons



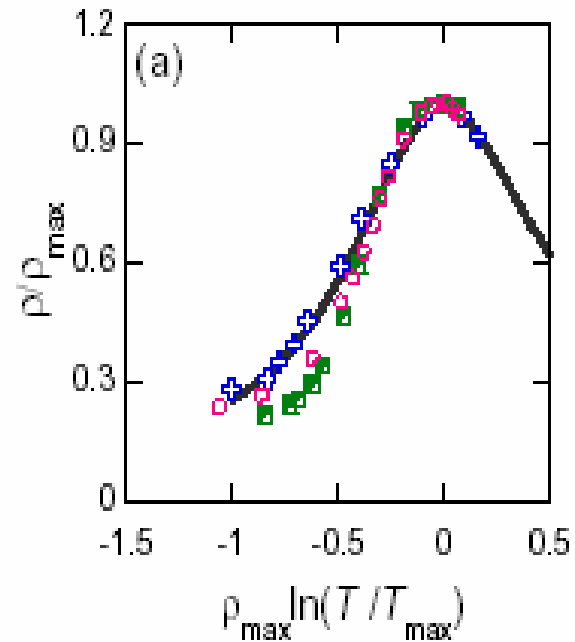
Flow diagram

A. Punnoose and A. Finkel'stein,
Science **310**, 289 (2005)

Si-MOSFET



*Comparison between
theory (lines)
and
experiment (symbols from the left plot)*



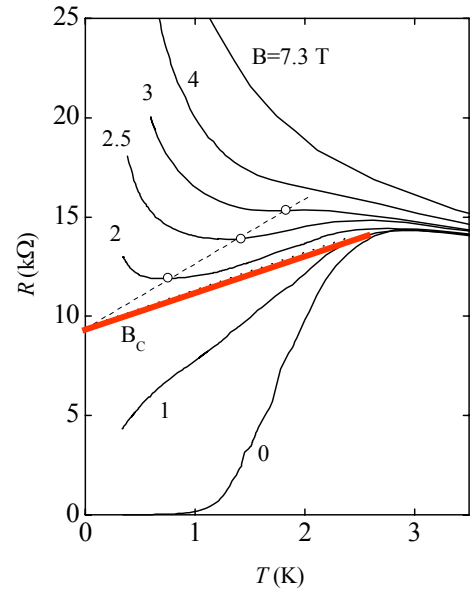
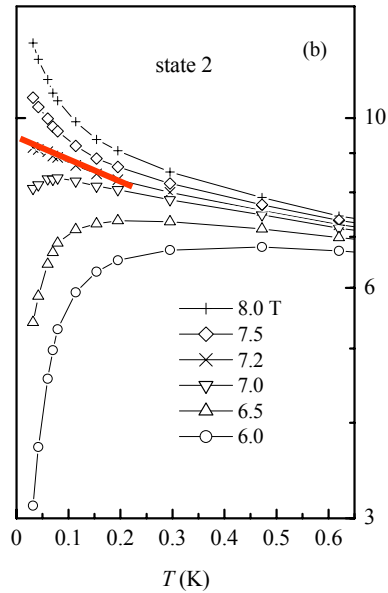
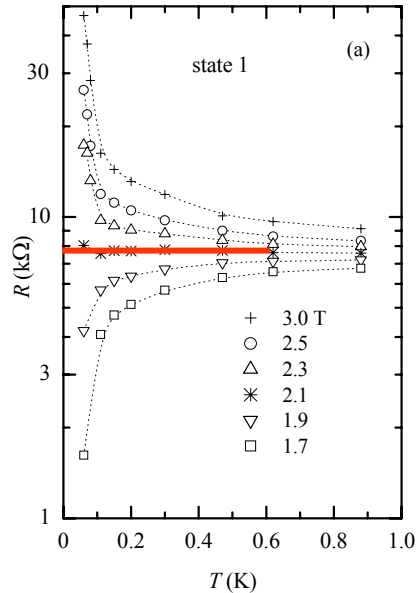
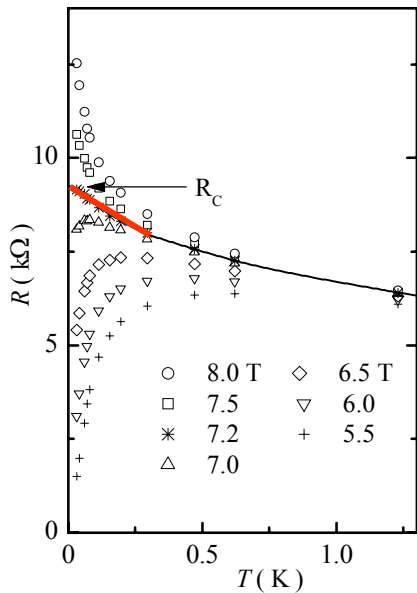
S.Anissimova, S.V.Kravchenko, A.Punnoose, A.Finkel'stein, and T.M.Klapwijk
cond-mat/0609181

M.P.A. Fisher, PRL 65, 923 (1990)

Phenomenological description in the frame of QPT
of 2D transitions based on $2e$ -bosons – vortex duality

Predicts scaling relations with $|B - B_{c0}| / T^{1/z\nu}$ as scaling variable

Superconductor-insulator transition in In-O



The separatrix may have non-zero slope

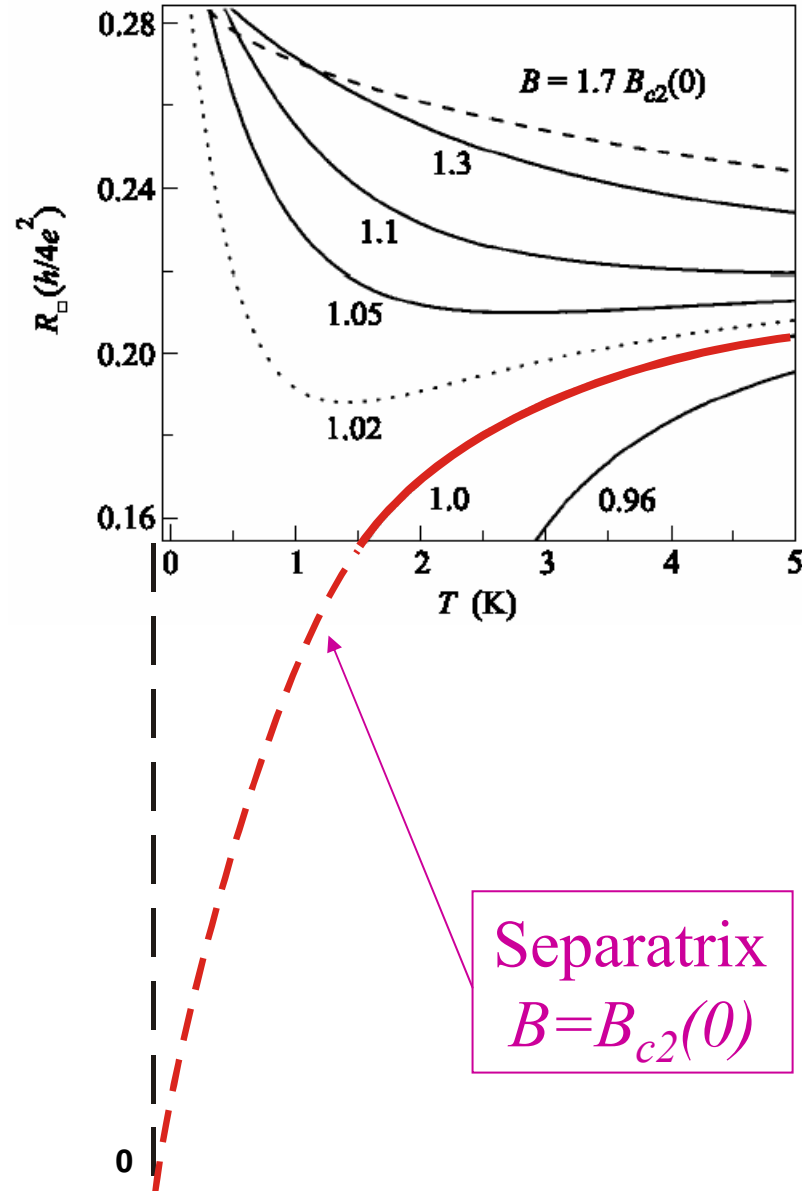
*V.M. Galitski and A.I. Larkin,
Phys.Rev. B 63, 174506 (2001)*

Superconducting fluctuations

at $T \ll T_{c0}$
in magnetic field

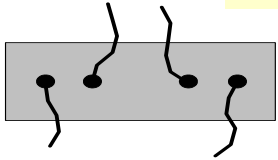
*Rigid calculations of the
superconducting
fluctuations*

How the magnetic field
destroys
the superconducting state
(dirty limit).

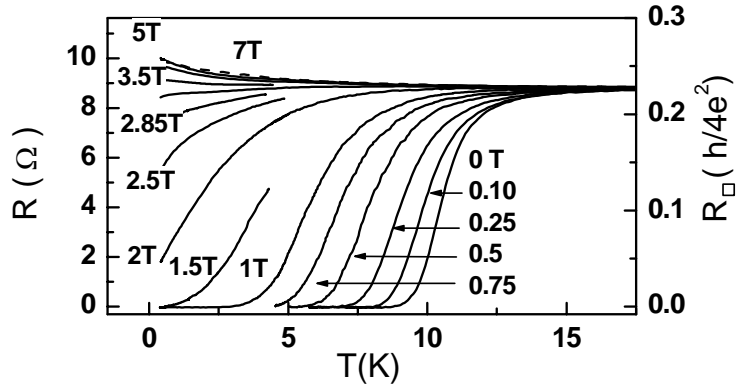


Film $\text{Nd}_{2-x}\text{Ce}_x\text{CuO}_{4+y}$

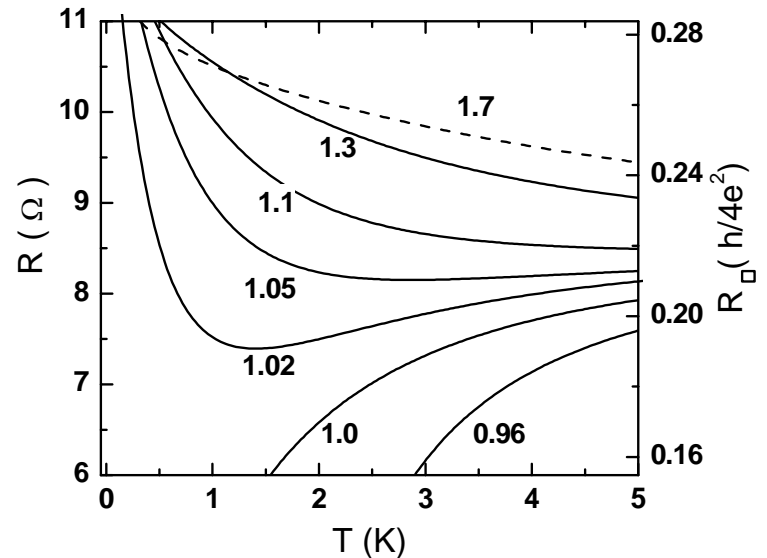
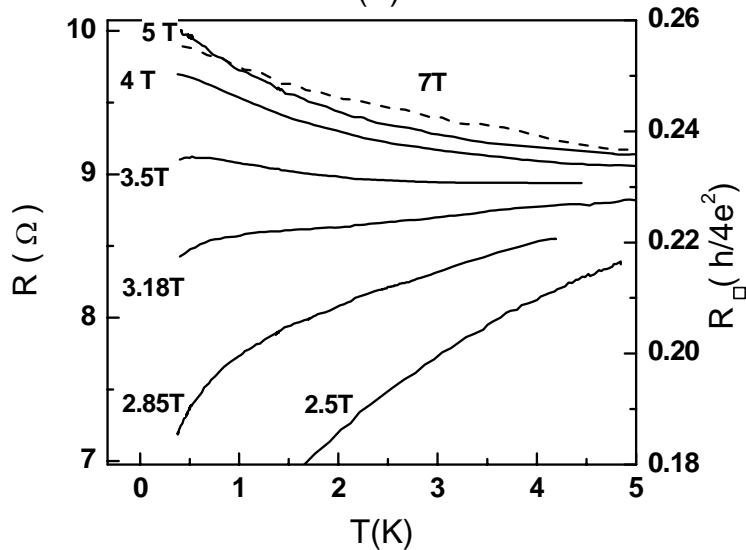
V.F. Gantmakher et al., JETPL 77, 424 (2003)



$d = 1000 \text{ \AA}$, $c = 12 \text{ \AA}$, $T_C = 11.5 \text{ K}$, $\Delta T = 2 \text{ K}$, $\sigma_0 = 17 e^2/h$



$$R(B, T) = 1 / [\sigma_0 - \alpha \frac{e^2}{h} \ln(T) + \Delta\sigma_{fl}(B, T)]$$



EXPERIMENT

CALCULATION

Conclusions

1. Metal-insulator transitions

3D-system of non-interacting electrons

2D-system of non-interacting electrons

2D-system of electrons with spin-orbit interaction

2D-system of interacting electrons

There is consensus between different theoretical approaches and between theory and experiment

2. Superconductor-insulator transition

Apparently, there are different scenarios but the main questions remain unresolved. Neither experiment, not theory have answers concerning 3D materials.