Fluctuation phenomena near the magnetic-field-tuned superconducting transition

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References:
- PRB 63, 174506 (2001), VG and A. I. Larkin
- PRB 67, 144501 (2003), VG and S.D. Sarma
Outline

• Introduction to Aslamazov-Larkin theory
• Quantum fluctuations near $H_{c2}(0)$
• Negative fluctuation magnetoresistance, $\delta\sigma \propto -\ln \frac{1}{(H-H_{c2})}$.
  Higher order diagrams.
• Bosonic and fermionic mechanisms of the negative MR
• Unusual metallic phase in disordered SC films and its possible exotic scenario: Fermionic vortices (theory formally equivalent to Ioffe-Larkin uniform RVB spin liquid).
Aslamazov-Larkin theory
Superconducting fluctuations

- At high temperatures, the mean field density of Cooper pairs is zero, \( \langle \Delta \rangle = 0 \): but fluctuating Cooper pairs can appear even above the transition. The order parameter fluctuates, \( \langle \Delta \Delta^* \rangle \neq 0 \).

Effects of fluctuations on transport:
- Direct conductivity of fluctuations (Aslamazov-Larkin, 1968)
- Decrease in the electron density (DOS)
- Scattering off the fluctuations (Maki-Thomson, 1970)
Fluctuation conductivity near $T_C$

- Drude conductivity of a normal metal

$$\sigma_0 = \frac{n ee^2 \tau}{m}$$

The number of Cooper pairs and their lifetime are singular quantities near the transition $\tau_{cp}(q) \propto N_{cp}(q) \propto \left( T - T_C + a q^2 \right)^{-1}$.

- Aslamazov-Larkin correction contains two singularities:

$$\delta \sigma_{AL} \propto \int d^2 q \frac{N_{cp}(q)(2e)^2 \tau_{cp}(q)}{2m} \propto \frac{T_C}{T - T_C}$$

- Density of states contains only one singularity:

$$\delta \sigma_{DOS} \propto -\int d^2 q \frac{2 N_{cp}(q)e^2 \tau_e}{m} \propto -\ln \left[ \frac{T_C}{T - T_C} \right]$$
Naïve derivation of diagrammatics for superconducting fluctuations

\[ \mathcal{L}(\omega, q) = \lambda + C(\omega, q) \]

\[ j^\alpha = \sigma_{\alpha\beta} E_\beta = -\frac{1}{i\omega} \sigma_{\alpha\beta} A_\beta, \quad \sigma_{\alpha\beta} \propto \frac{1}{i\omega \delta A^\alpha A^\beta} \bigg|_{A \to 0} \]

\[ \frac{\delta G}{\delta A} \bigg|_{A \to 0} \propto \frac{\delta}{\delta A i\varepsilon - (p - eA)^2 / (2m)} = \text{GeV} \]

\[ \frac{\delta^2 \Omega}{\delta A^\alpha A^\beta} = \frac{\delta^2}{\delta A^\alpha A^\beta} \]

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Aslamazov-Larkin result

\[ \mathcal{L}(\omega_n, q) \propto \frac{1}{\gamma|\omega_n| + aq^2 + (T - T_c)} \]

AL result in two dimensions

\[ \delta\sigma = \frac{e^2}{16\hbar} \frac{T_c}{T - T_c} \]
Experimental verification of the Alsamazov-Larkin theory


The region where the mean-field theory breaks down near the transition (Ginzburg region):

\[
\frac{T - T_c}{T_c} < \text{Gi} = \begin{cases} 
T_c/E_F, & \text{clean system} \\
(E_F \tau)^{-1}, & \text{disordered system}
\end{cases}
\]
Quantum SC fluctuations near $H_{c2}(0)$
Suppression of $T_C$

- Strong fields; Gapless excitations (de Gennes):
- Weak or zero field; Gap in the spectrum (BCS):

$$E(p) = \sqrt{\xi^2(p) + \Delta^2}$$
Unusual fluctuations near $H_{c2}(0)$

Mo$_x$Si$_{1-x}$: S. Okuma et al. (1998)  
TiN$_x$: V. F. Gantmakher et al. (2002)
The usual Ginzburg-Landau theory is based on a long wavelength expansion, but in strong fields, \((p - eA)^2\) is not in any sense small.

\[
S[\Delta] = \int_{1,2} \Delta_1^* A(1,2) \Delta_2 + \int_{1,2,3,4} \Delta_1^* \Delta_2^* B(1,2,3,4) \Delta_3 \Delta_4
\]

\(\tilde{A}\) is a non-local integral operator, but it is diagonal in the Landau basis (Helfand & Wethamer):

\[
\langle n | \tilde{A} | m \rangle = A_n \delta_{n,m}
\]

The transition point is determined by the condition

\[
\mathcal{L}_{n=0}(\omega = 0) = A_0^{-1} = \infty
\]
Fluct. propagator is a diagonal operator in the Landau basis, $\mathcal{L}(\omega)$.

- At the LLL its matrix element is singular

$$\mathcal{L}_0(\omega) \propto \frac{1}{\gamma|\omega| + [H - H_{c2}(0)]/H}$$

- At all higher levels, $\mathcal{L}_{n>0}(\omega = 0) = \text{const} < \infty$

The conductivity has velocity operators in the vertices

$$\langle n | \tilde{v}_\alpha | m \rangle \propto \delta_{n+1,m}, \text{ thus}$$

Aslamazov-Larkin  Density of States  Maki-Thomson
Naïve qualitative estimate of the fluctuation conductivity
\[ \delta \sigma \propto \int d\omega N_{cp}(\omega) = \int \frac{d\omega}{\gamma |\omega| + (H - H_{c2})/H} \propto \ln \left[ \frac{H}{H - H_{c2}(0)} \right] \]

The sign of the correction is not obvious!
One needs to calculate all diagrams:

**Fluctuation conductivity**
\[ \delta \sigma_{\text{tot}} = -\frac{2e^2}{3\pi^2 \hbar} \ln \left[ \frac{H}{H - H_{c2}(0)} \right] \]

**Fluctuation magnetization**
\[ \delta \chi = \frac{2e^2 D}{\pi^2 \hbar c^2} \frac{H}{H - H_{c2}(0)} \gg \chi_{\text{Landau}} \]
Quantum-to-classical crossover
Experimental verification

General expression at low but finite temperatures

$$\delta\sigma = \frac{2e^2}{3\pi^2\hbar} \left\{ -\ln\frac{r}{\hbar} - \frac{3}{2r} + \psi(r) + 4\left[ r\psi'(r) - 1\right] \right\}, \quad r = \frac{1}{2\gamma} \frac{H - H_{c2}(0)}{H} \frac{T_{c0}}{T}$$

Quant.-to-class. crossover

$$\delta\sigma \propto \begin{cases} [T - T_c(H)]^{-1}, & \text{if } t \gg h; \\ -\ln\frac{H}{H-H_{c2}(0)}, & \text{if } t \ll h. \end{cases}$$

Baturina et al., Physica B 359, 500 (2005)
How to define the Ginzburg region? Compare the leading and sub-leading terms.

- At $H = 0$, $(T - T_{c0})/T \gg \xi \sim 1/(E_F \tau)$
- At $T = 0$, $[H - H_{c2}(0)]/H \gg \xi$
- In the intermediate $T$-regime, we have $[H - H_{c2}(T)]/H \gg \sqrt{\xi}$
Is it possible to collect the logs within an RG/parquet scheme?

Corrections to the quadratic coefficient $A$ of GL:

$$F[\Delta] = \tilde{A}\Delta^2 + \tilde{B}\Delta^4$$

\[ \propto G_i \log \left( H - H_{c2} \right) \]

\[ \propto G_i^2 \log^2 \left( H - H_{c2} \right) \]

\[ \propto G_i^3 \frac{\log \left( H - H_{c2} \right)}{H - H_{c2}} \]
An earlier “dirty-boson” theory
\[ \hat{\mathcal{H}} = -\rho_s \sum_{\langle ij \rangle} \cos (\phi_i - \phi_j - A_{ij}) + U \sum_i \hat{n}_i^2 \]

It is possible to re-write the theory in terms of vortices (defects in the phase field): \( \hat{b}_v^+ \) creates a vortex and \( \hat{b}_v \) annihilates a vortex.

\[ \hat{b}_v = \sqrt{\hat{n}_v} e^{i\hat{\theta}}, \quad [\hat{n}_v, \hat{\theta}] = i \]

**Dual Hamiltonian**

\[ \hat{\mathcal{H}} = -t_V \sum_{\langle ij \rangle} \cos (\hat{\phi}_i - \hat{\phi}_j - a_{ij}) + \frac{1}{2} \sum_{ij} (\hat{n}_v,i - B) V_{ij} (\hat{n}_v,j - B) + \ldots \]

- **Vortex hopping**
- **Dual gauge field**
- **Cooper pair density**
- **Inter-vortex interaction** \( \sim \ln R_{ij} \)
- **Gauge field fluctuations**
Boson-vortex duality

Field seen by Cooper pairs → Density of vortices
Field seen by vortices → Density of cooper pairs
Cooper pair condensation → Vortex localization
Vortex condensation → Cooper pair localization

Vortex conductivity = Particle resistivity
Schematic phase diagram


- Superconducting phase has delocalized pairs and localized vortices.
- Insulating phase has localized pairs and delocalized vortices.
- The phase boundary is a “Bose metal”.
Which theory describes the experiment?

In Ann. Phys. (Leipzig) 8, 785 (1999), A. I. Larkin posed a similar question in the context of disorder-induced SC-INS transition and his suggestion was:

- If $G_i \ll 1$, the "fermionic theory" works
- If $G_i \gg 1$, the "bosonic theory" works
Unusual metallic phase in dirty SC films

and its possible theoretical scenario
MoGe films:

Metallic phase

Ta films:
(Jongsoo Yoon)
Our system includes:
- Cooper pairs
- Gapless excitations
- A magnetic field
- Disorder
In dual language:

- Quantum vortices
- Gapless excitations
- Dual magnetic field acting on the vortices
- Disorder

If vortices are bosons they are either localized or condensed at zero temperature!
Composite vortices

- The dual magnetic field acting on the vortices is related to the densities of Cooper pairs and quasiparticles:
  \[ B_{\text{dual}} = 2\pi N_{\text{CP}} - \pi N_{\text{qp}} \]

- In a disordered SC, the dual field is not known but it can be very large ("dual quantum Hall" physics is possible).

- Formation of composite particles (bound state of a vortex and fluxes) is possible.

\[ \text{Boson + 1 flux = fermion} \quad \text{Boson + 2 fluxes = boson} \]
Field theory for fermionized vortices

\[ \mathcal{L} = \psi^\dagger \left[ -\frac{1}{2m_\psi} (\nabla - ia + iA)^2 + (\partial_\tau - ia_0 + iA_0) \right] \psi \\
+ \frac{1}{2C} (\nabla \times a)^2 + \frac{1}{4\pi} A \nabla \times A \]

Let us shift variables \( a_\mu - A_\mu \rightarrow a_\mu \) and integrate out the Chern-Simons field:

\[ \mathcal{L} = \psi^\dagger \left[ -\frac{1}{2m_\psi} (\nabla - ia)^2 + (\partial_\tau - ia_0) \right] \psi \\
+ \frac{1}{2C} (\epsilon_{\mu\nu\lambda} \partial_\nu a_\lambda)^2 + \frac{1}{4C^2} (\nabla \times a) \cdot (\nabla \times \nabla \times a) \]

The Chern-Simons terms drop out of the action. Statistics is less important than interactions!!!
The appropriate language is a gauge theory.

\[ \mathcal{L} = \psi^\dagger \left[ -\frac{1}{2m_v} (\nabla - ia)^2 + (\partial_r - ia_0) \right] \psi + \frac{1}{2c} \left( \epsilon_{\mu\nu\lambda} \partial_{\nu} a_{\lambda} \right)^2 + \text{disorder} \]

Integrate out diffusive fermions:

\[ \mathcal{E} \sim \int d\omega d^2q \left[ \varepsilon(\omega, q)E^2 + \mu B^2 \right] \]

Electric and magnetic fields:

\[ E_i = -\partial_0 a_i - \partial_i a_0, \quad B = \text{curl} a \]

Diffusive fermionic vortices lead to the dielectric constant:

\[ \varepsilon(\omega, q) \propto |\omega|^{-1} \]

It is an anisotropic \((2 + 1)\) electrodynamics.
Tunneling into the vortex metal

- Tunneling conductance:

$$G \propto \int_0^\infty dt \langle [I(t), I(0)] \rangle, \quad I(t) = 2eJ \sin \phi(t)$$

- We need to calculate the correlation function:

$$C(t) = \langle e^{-i\phi(0,0)} e^{i\phi(0,t)} \rangle$$

In the language of the (2+1)-electrodynamics, it means inserting a monopole at the origin and anti-monopole at $y(0, t)$.

- Energy of this configuration:

$$\mathcal{E} \sim \sigma_v \ln^2 (t/\tau)$$

- Tunneling conductance:

$$G(T) \propto \exp \left[ -\frac{\sigma_v}{2} \ln^2 (T \tau) \right]$$
Summary

- Quantum superconducting fluctuations near $H_{c2}(0)$ lead to negative magnetoresistance in two dimensions.
- The Ginzburg region becomes wider at intermediate fields/temperatures.
- The bosonic and fermionic theories are not necessarily competing theories. Their applicability is controlled by the width of the fluctuation region.
- A possible scenario of a low-temperature metallic phase in dirty superconducting films is a statistical transmutation in the system of vortices, which may be converted into fermions.