Fractal Superconductivity near Localization Threshold

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Short publication: Phys Rev Lett. **98**, 027001 (2007)

Superconductity v/s Localization

- Granular systems with Coulomb interaction

K.Efetov 1980 et al

- Coulomb-induced suppression of Tc in uniform films

A.Finkelstein 1987 et al

- Competition of Cooper pairing and localization (no Coulomb)

Imry-Strongin, Ma-Lee, Kotliar-Kapitulnik, Bulaevsky-Sadovsky(mid-80's)

Ghosal, Randeria, Trivedi 2001

There will be no grains and no Coulomb in this talk !

Plan of the talk

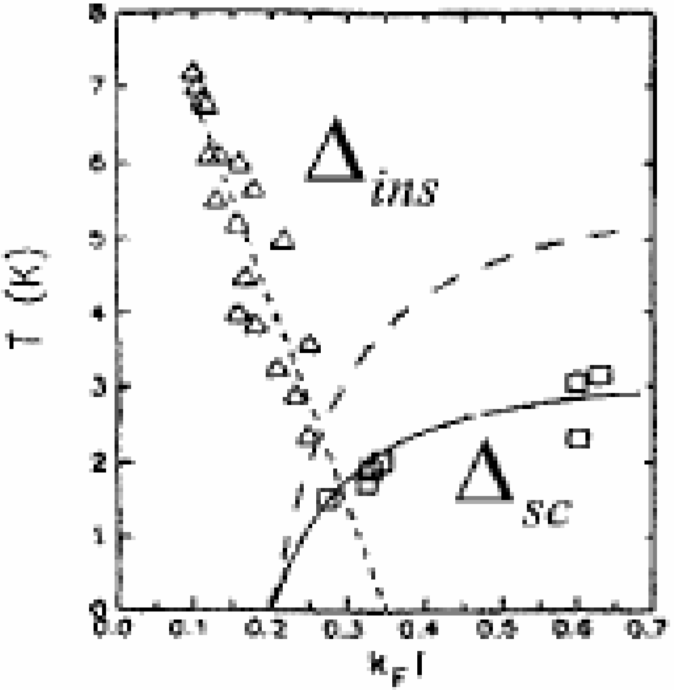
1. Motivation from experiments

2. Hard-gap insulator due to electron pairing on localized states

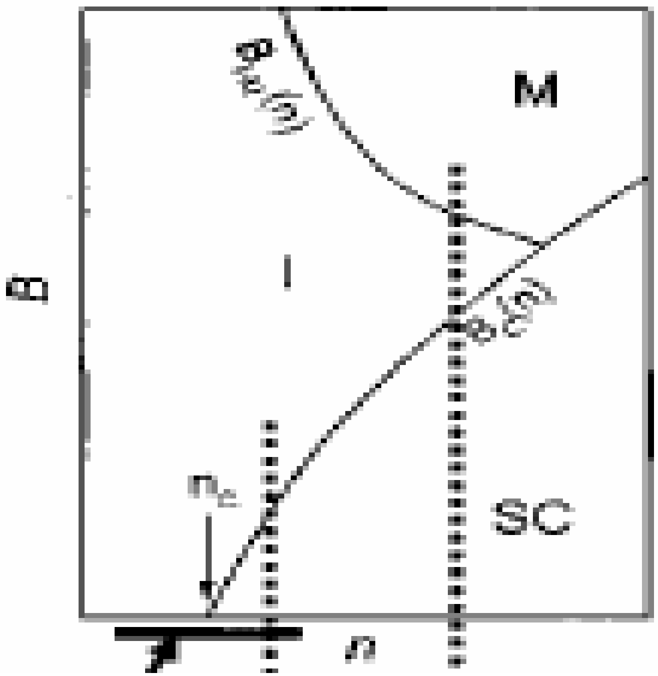
3. BCS-like theory for critical eigenstates

4. S-I transition region and pseudogap

Experimental puzzle: Localized Cooper pairs

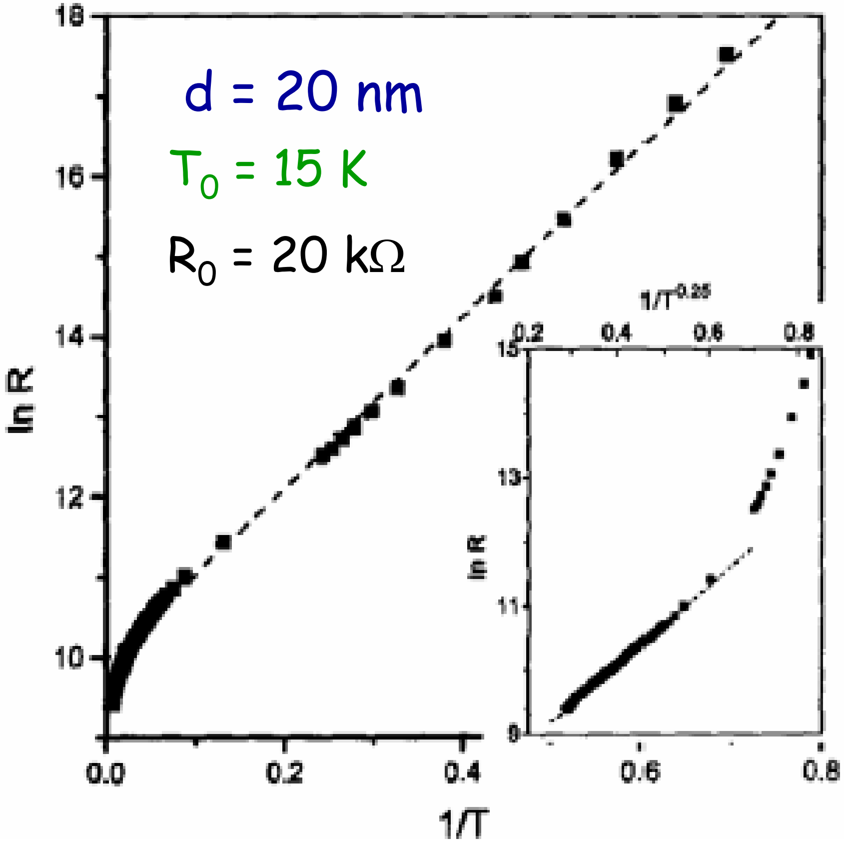


D.Shahar & Z.Ovadyahu
amorphous InO 1992

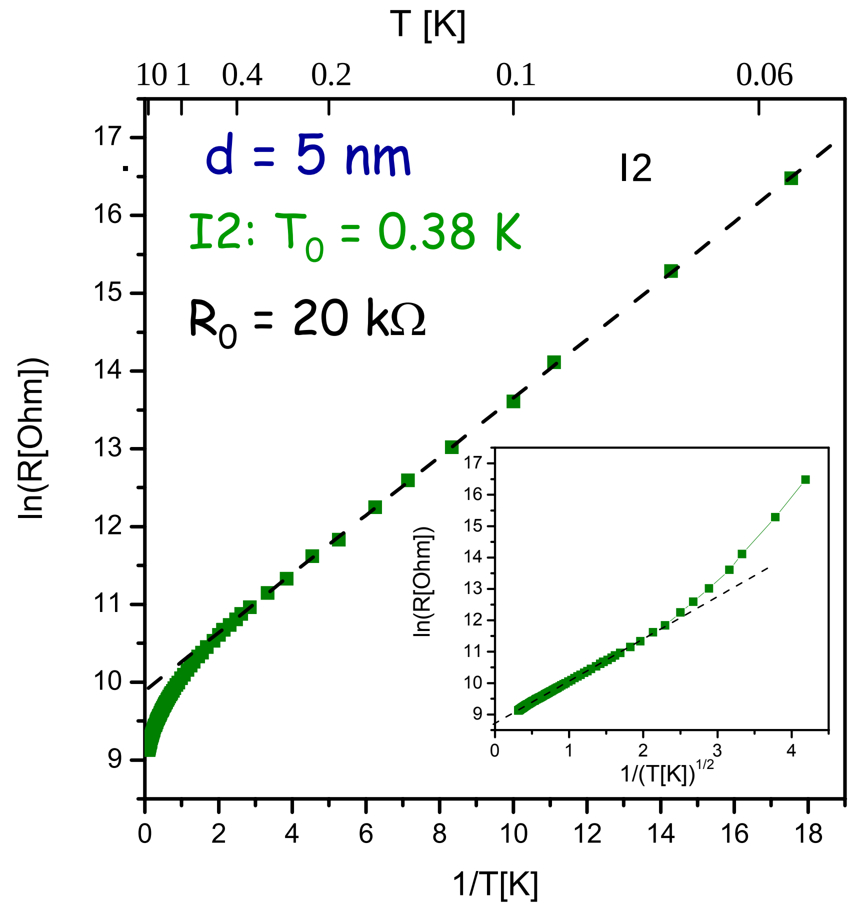


V.Gantmakher et al InO
D.Shahar et al InO
T.Baturina et al TiN

Strongly insulating InO and nearly-critical TiN

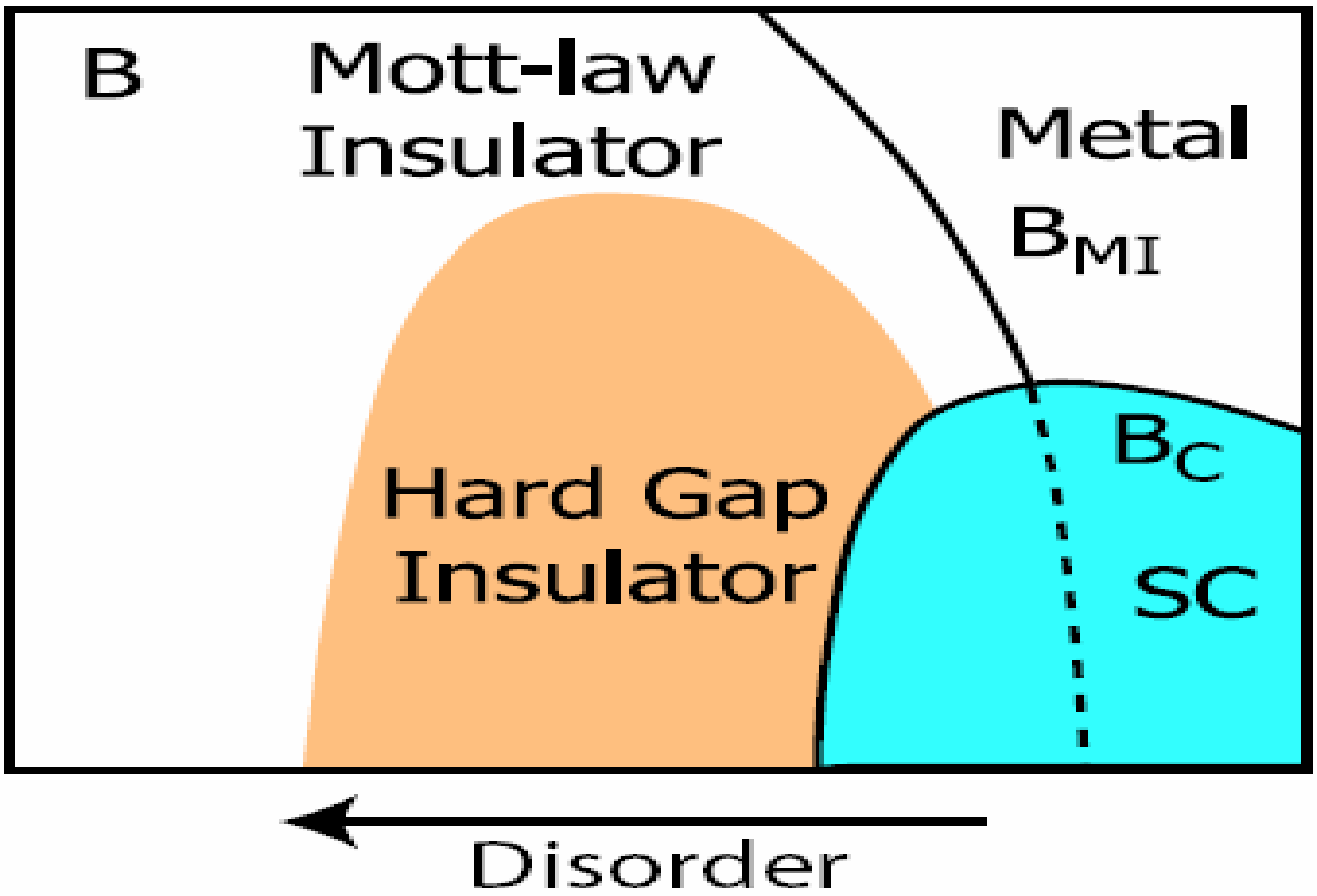


Kowal-Ovadyahu 1994



Baturina et al 2007

Phase Diagram



Theoretical model

Simplest BCS attraction model,

but for localized electrons

$$H = H_{kin} - g\, \Psi_{\uparrow}^{\dagger} \Psi_{\downarrow}^{\dagger} \Psi_{\downarrow} \Psi_{\uparrow}$$

$$\boldsymbol{\Psi} = \Sigma\, c_j\, \Psi_j\,(r)$$

Basis of localized eigenfunctions

M. Ma and P. Lee (1985)

Localization length L_{loc} is **finite but large**

$$H = \sum_{j\sigma} \xi_j c^\dagger_{j\sigma} c_{j\sigma} - \frac{\lambda}{\nu_0} \sum_{i,j,k,l} M_{ijkl} c^\dagger_{i\uparrow} c^\dagger_{j\downarrow} c_{k\uparrow} c_{l\downarrow} ,$$

where

$$M_{ijkl} = \int d\mathbf{r} \psi_i(r) \psi_j(r) \psi_k(r) \psi_l(r)$$

$$\lambda = g\nu_0$$

$$M_{ij} = \int \psi_i^2(r) \psi_j^2(r) d^d r$$

$$M_j = \int \psi_j^4(r) d^d r \propto L_{loc}^{-D_2}$$

$D_2 = 1.30 \pm 0.05$

All other (off-diagonal) terms: beyond BCS MFA

INSULATING STATE AT LARGE $\delta_L \equiv (\nu_0 L_{loc}^3)^{-1}$

Typical value of superdiagonal matrix element:

$$\bar{M} = L_0^{-3}(L_{loc}/L_0)^{-D_2}$$

where L_0 is the short-scale cutoff length of the fractal behaviour.

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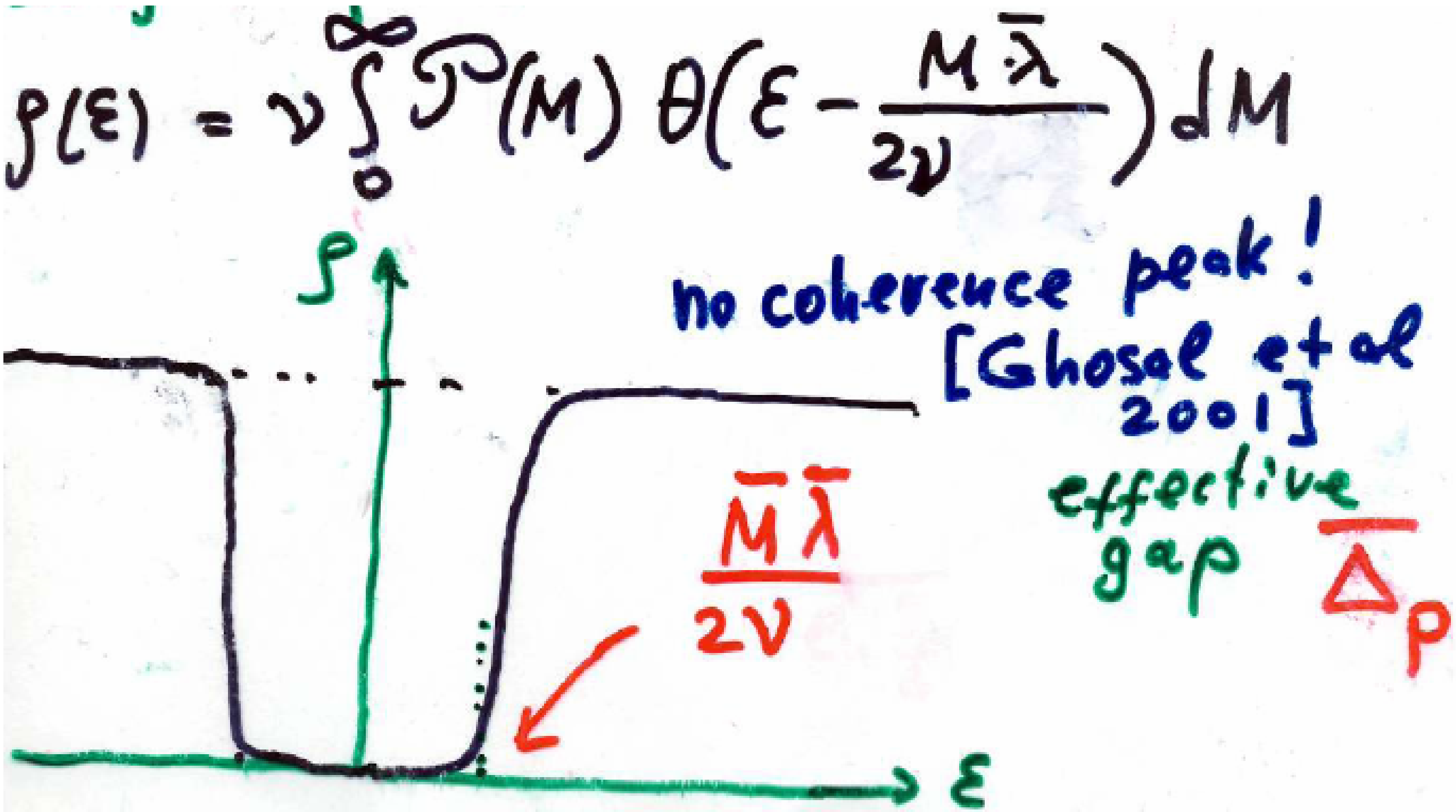
$$\Delta_P = \frac{\lambda}{2} E_0 \left(\frac{L_0}{L_{\text{loc}}}\right)^{D_2} \propto (E_m - E_F)^{\nu D_2}$$

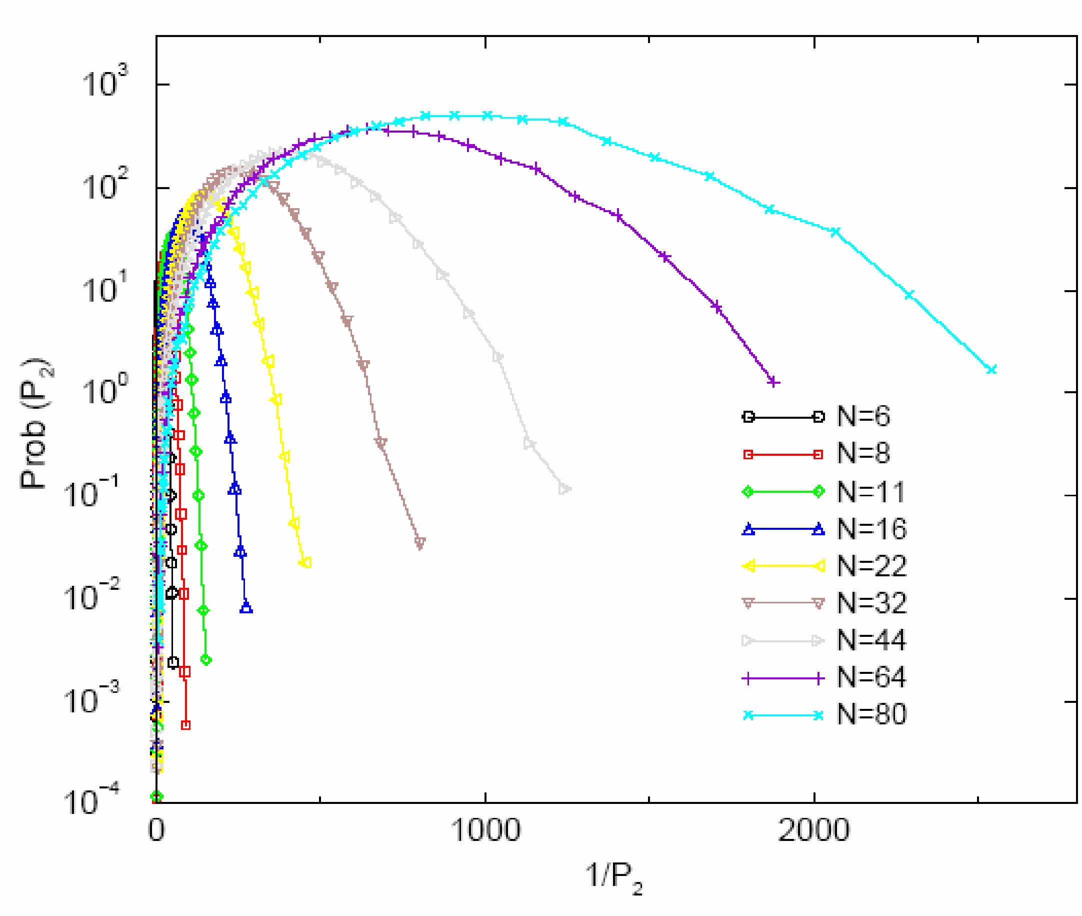
where

$$E_0 = \frac{1}{\nu_0 L_0^3} \ll E_F$$

Average Density of States



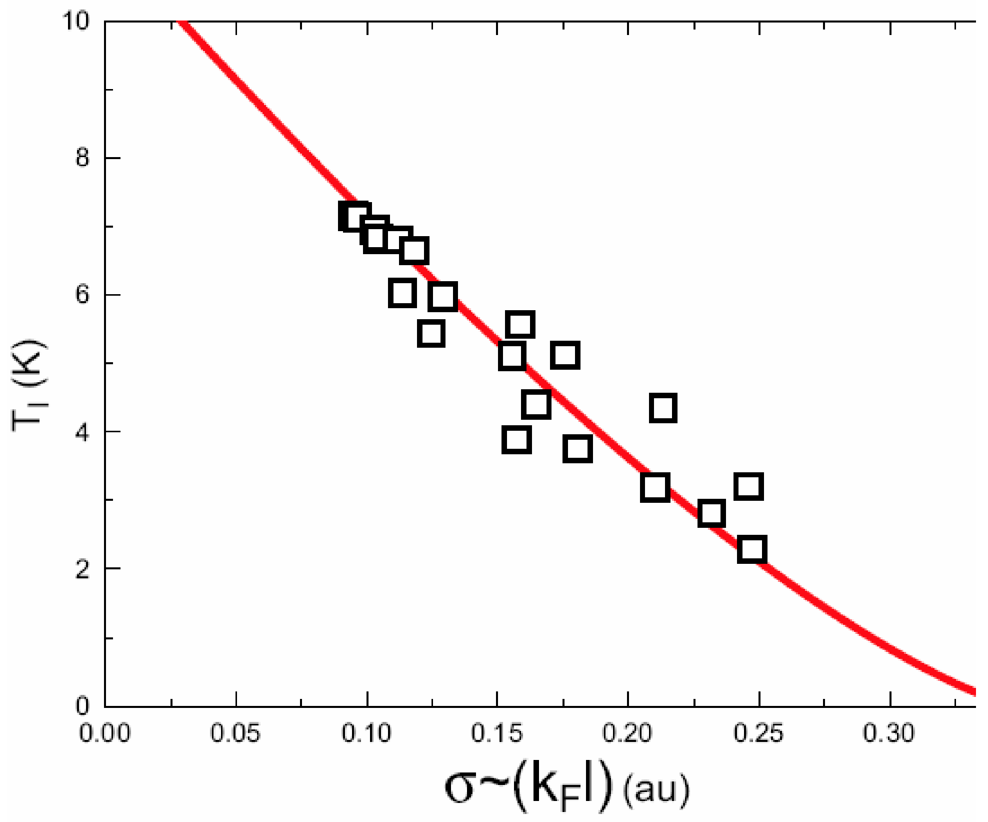
P(M) distribution



A. Mildenberger and A.. Mirlin, private correspondence

(Critical Ensemble)

Activation energy T_I from Shahar-Ovadyahu exp. and fit to theory



Superconductivity at the Localization Threshold: $T_c >> \delta_L$

Now we will consider the case of Fermi energy very close to the mobility edge:

single-electron states are extended but **fractal**

and populate small fraction of the whole volume

How BCS theory should be modified to account for eigenstates fractality ?

Mean-Field Eq. for T_c

$$\Delta(r) = \int K_T(r, r')\Delta(r')d^d r' \tag{9}$$

where kernel $\hat{K}_T$ is equal to

$$K_T(r, r') = \frac{\lambda}{2\nu_0}\sum_{ij}\frac{\tanh\frac{\xi_i}{2T} + \tanh\frac{\xi_j}{2T}}{\xi_i + \xi_j}\psi_i(r)\psi_j(r)\psi_i(r')\psi_j(r') \tag{10}$$

Standard averaging over space $\Delta(r) \to \overline{\Delta}$ leads to "Anderson theorem" result: totally incorrect in the present situation.

The reason: critical eigenstates $\psi_j(r)$ are strongly correlated in real 3D space, they fill some small **submanifold** of the whole space only.

In fact one should define T_c as the divergence temperature of the Cooper ladder

$$\mathcal{C} = \left(1 - \hat{K}\right)^{-1}$$

Thus averaging procedure should be applied to $\mathcal{C}$ instead of K

We expand $\mathcal{C}$ in powers of K and average over disorder realizations. Keeping main sequence of resulting diagramms only, we come to the following equation for determination of T_c:

$$\Phi(\xi) = \frac{\lambda}{2} \int \frac{d\xi' \tanh(\xi'/2T)}{\xi'} M(\xi - \xi') \Phi(\xi') \tag{11}$$

$$M(\omega) = \mathcal{V}\overline{M_{ij}} = \int \overline{\psi_i^2(r)\psi_j^2(r)} d^d r \quad \text{for} \quad |\xi_i - \xi_j| = \omega$$

For critical eigenstates

$$L_{\text{loc}} \to \infty$$

one finds

$$M(\omega) = \left(\frac{E_0}{\omega}\right)^\gamma$$

where

$$\gamma = 1 - \frac{D_2}{d}$$

is a measure of fractality

Usual "dirty superconductor":

$$M(\omega) = 1 \qquad \gamma = 0$$

3D Anderson model: **γ** = 0.57

The above equation for T_c is equivalent to the neglect in the Hamiltonian "off-diagonal" terms. We employ eigenfunction expansion of the gap function $\Delta(r)$ and use the idea that pairing amplitude

$$F_j = \langle c_{j\uparrow} c_{j\downarrow} \rangle = F(\xi_j)$$

is a smooth function of the bare energy ξ_j:

$$F(\xi) = \frac{\Delta(\xi)}{\sqrt{\Delta^2(\xi) + \xi^2}} \tanh \frac{\sqrt{\Delta^2(\xi) + \xi^2}}{2T}$$

where

$$\Delta(\xi) = \lambda \int d\xi' M(\xi - \xi') F(\xi')$$

Then local pairing amplitude:

$$F(r) = \sum_j \psi_j^2 \langle c_{j\uparrow} c_{j\downarrow} \rangle \equiv \sum_j \psi_j^2 F_j$$

fluctuates strong in real space

Volume fraction $(T_c/E_0)^\gamma << 1$

Self-consistent "gap equation" in terms of $\Delta(\xi)$:

$$\Delta(\xi) = \lambda \int d\xi' M(\xi - \xi') \frac{\Delta(\xi)}{\sqrt{\Delta^2(\xi) + \xi^2}} \tanh \frac{\sqrt{\Delta^2(\xi) + \xi^2}}{2T}$$

Dimensional analysis of the Mean Field equation:

$$T_c = C(\gamma) E_0 \lambda^{1/\gamma}$$

$$\Delta(\xi = 0, T = 0) = D(\gamma) E_0 \lambda^{1/\gamma}$$

Functions $C(\gamma)$ and $D(\gamma)$ were found numerically:

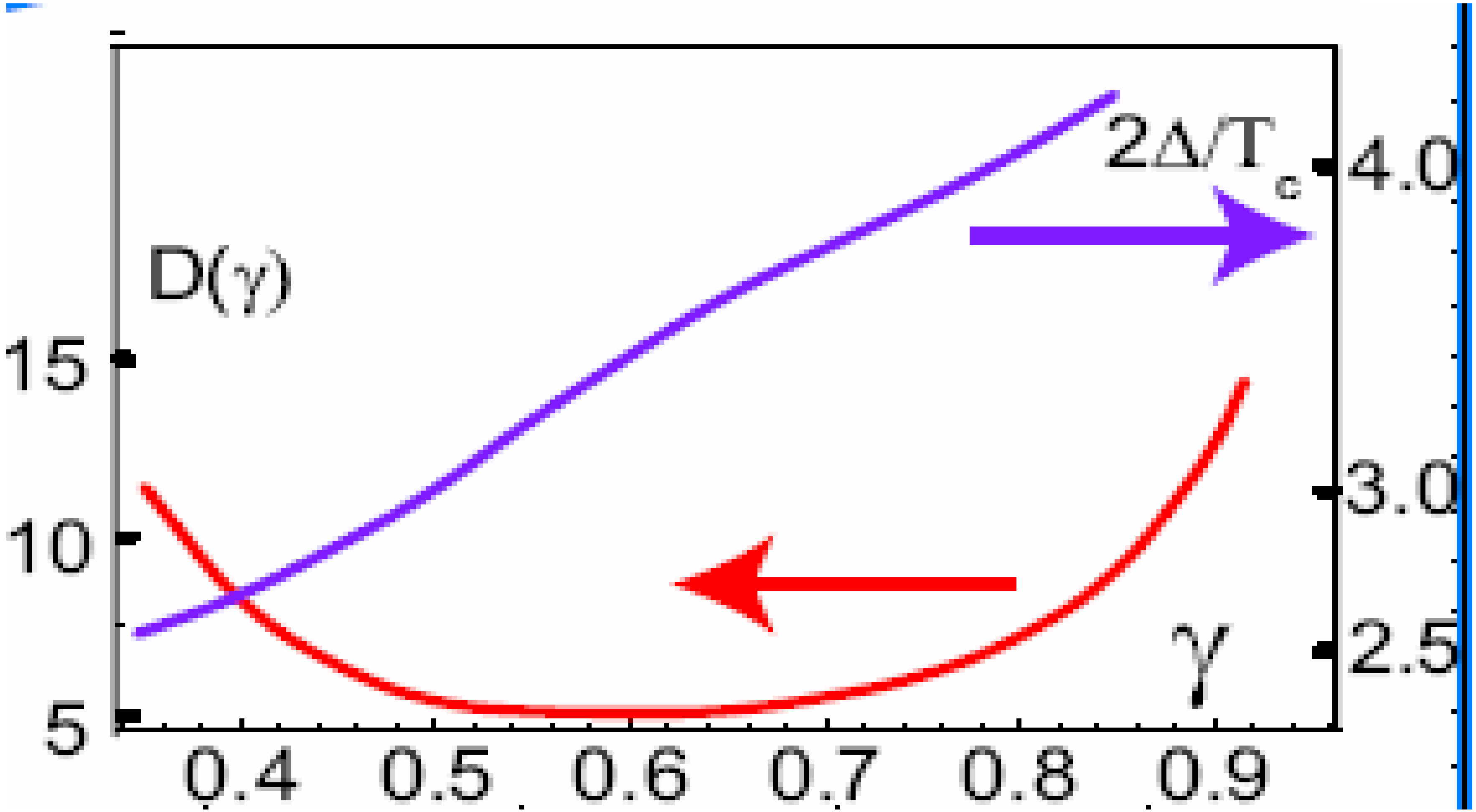
Now we can relate **collective gap** $\Delta(0)$ and **local pairing gap** Δ_P:

$$\Delta_P = \frac{1}{2D^\gamma(\gamma)} \delta_L \left(\frac{\Delta(0)}{\delta_L} \right)^\gamma$$

where $\delta_L = \frac{1}{\nu_0 L_{\text{loc}}^3}$ - typical level spacing inside localization volume .

Compare to Matveev-Larkin "parity gap"

$$\Delta_P = \frac{\delta}{2\log(\delta/\Delta)}$$



Comparison with virial expansion result: disagreement by 10%

I. VERTEX CORRECTIONS

Matrix elements of Cooper interaction are in general

$$M_{ijkl} = \int d^3r \psi_i(r)\psi_j(r)\psi_k(r)\psi_l(r)$$

Quantities M_{ijkl} are random with zero mean, In the second order

$$\delta\left[\lambda M(\epsilon_1 - \epsilon_2)\right] = \lambda^2 \nu_0 \int\int d\epsilon_1 d\epsilon_2 R(\epsilon_i, \epsilon_j, \epsilon_1, \epsilon_2) \cdot \frac{\tanh(\beta\epsilon_1/2) - \tanh(\beta\epsilon_2/2)}{2(\epsilon_1 - \epsilon_2)}$$

$$R(\epsilon_i, \epsilon_j, \epsilon_k, \epsilon_l) = \mathcal{V}^3 \overline{M_{ijkl}^2} \sim L_0^3 \left(\frac{E_0}{\omega}\right)^t$$

$$t = 3(1 - d_4/d)$$

and d_4 is the exponent entering expression $P_4 = \int dr(|\psi|^2(r))^4 = \mathcal{V}^{-3d_4/d}$.

$$\delta[\lambda M(\omega)] = \lambda^2 \left(\frac{E_0}{\omega}\right)^{2-3d_4/d}$$

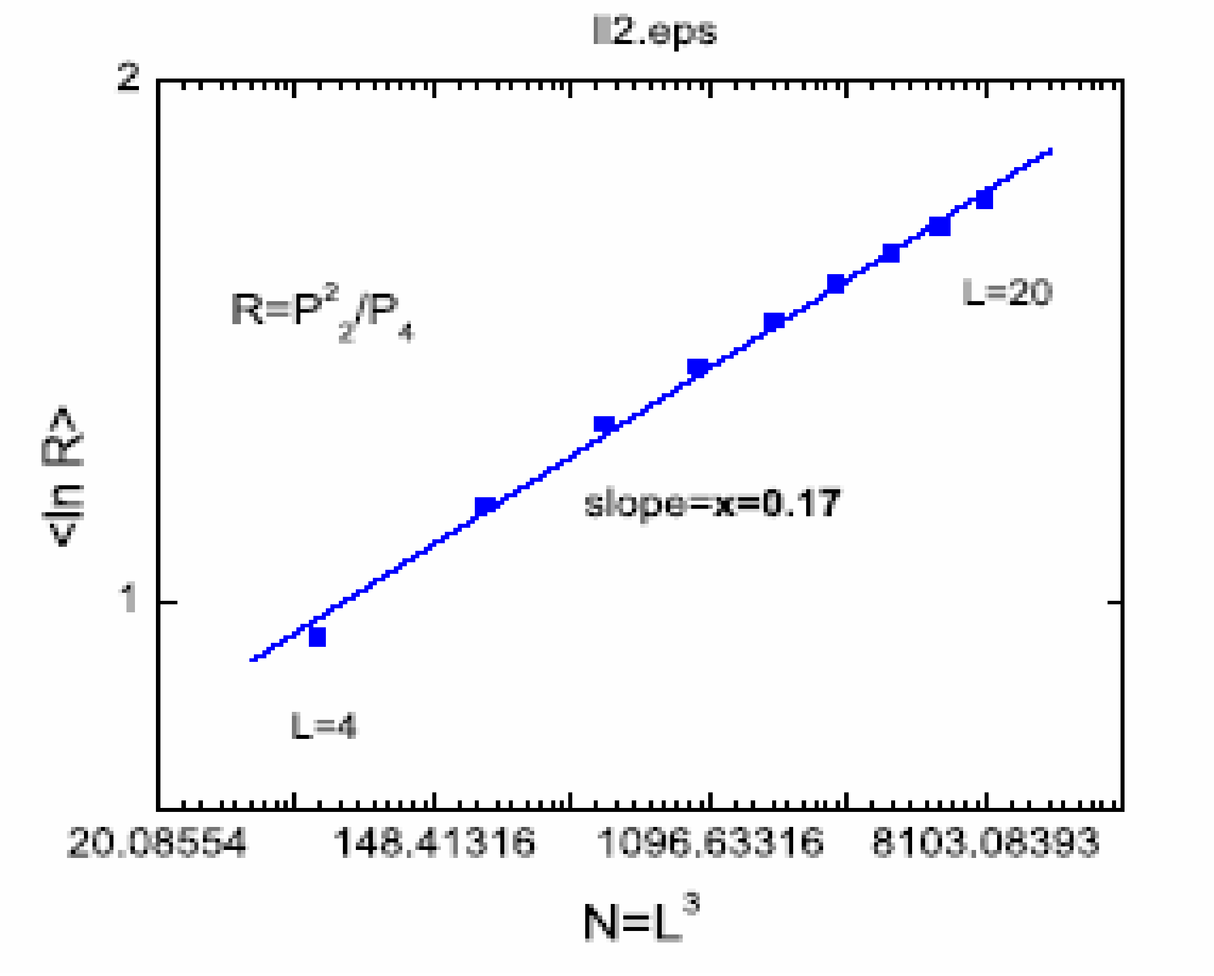
The condition $\delta[\lambda M(\omega)] \ll \lambda M(\omega)$ is fulfilled at $\omega \sim T_c$ if the exponent $x = d_4 - \frac{2}{3}d_2 > 0$, since their ratio scales as

$$\frac{\delta[\lambda M(T_c)]}{\lambda M(T_c)} \sim (T_c/E_0)^x$$

x > 0 always !

3D Anderson: x = 0.17

Numerics for 3D Anderson model:



Conclusion:
“diagonal” approximation makes sense, but corrections are non-negligible

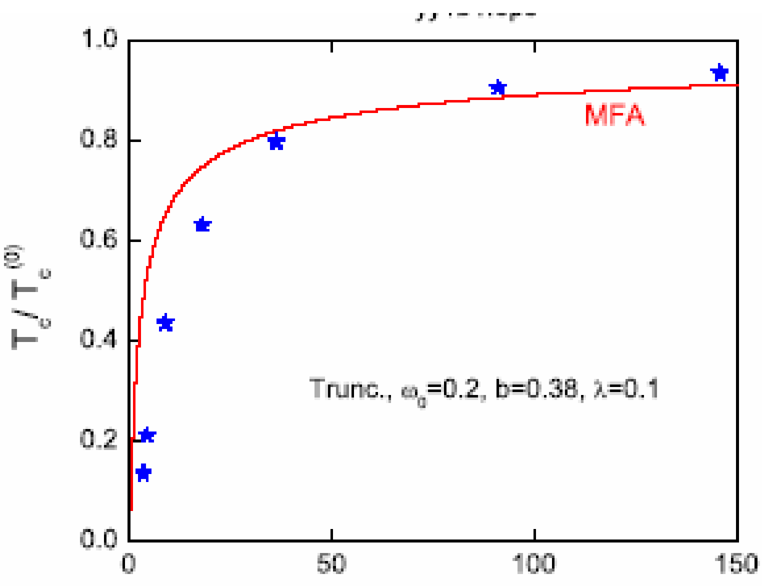
Go deeper into insulator

Level spacing δ_L becomes comparable to T_c

.

$$M(\omega) = \left(\frac{E_0}{|\omega| + \delta_L} \right)^{\gamma}$$

$$T_c = T_c^{(0)} \mathcal{T}_{\gamma} \left(\frac{T_c^{0}}{\delta_L} \right)$$



T_c / δ_L

However, this M(ω) is specific to 1D models only

.

3D Anderson insulator: **M(ω)**

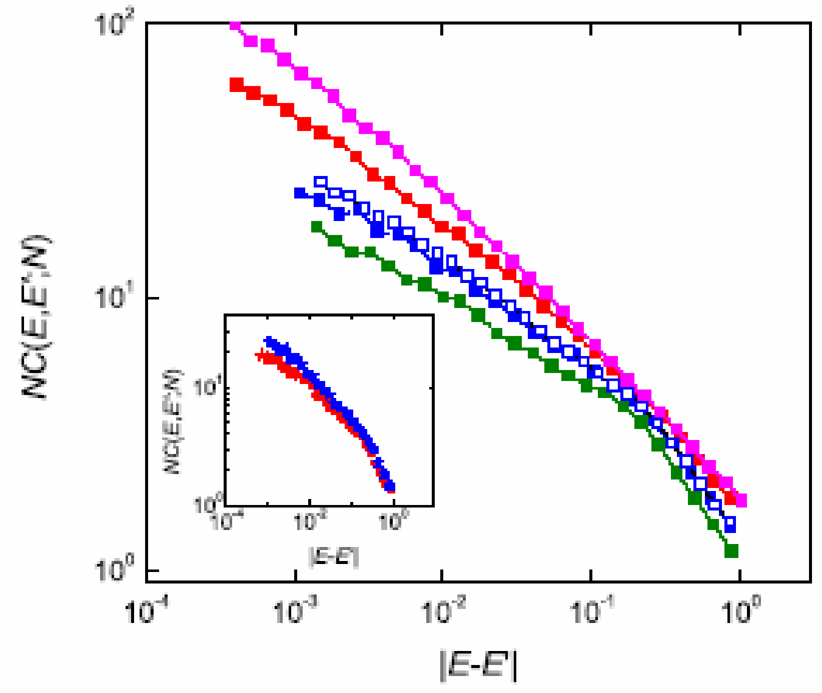
No saturation at $\omega < \delta_L$

Superconductivity with
$T_c < \delta_L$ is possible

Then "local gap"

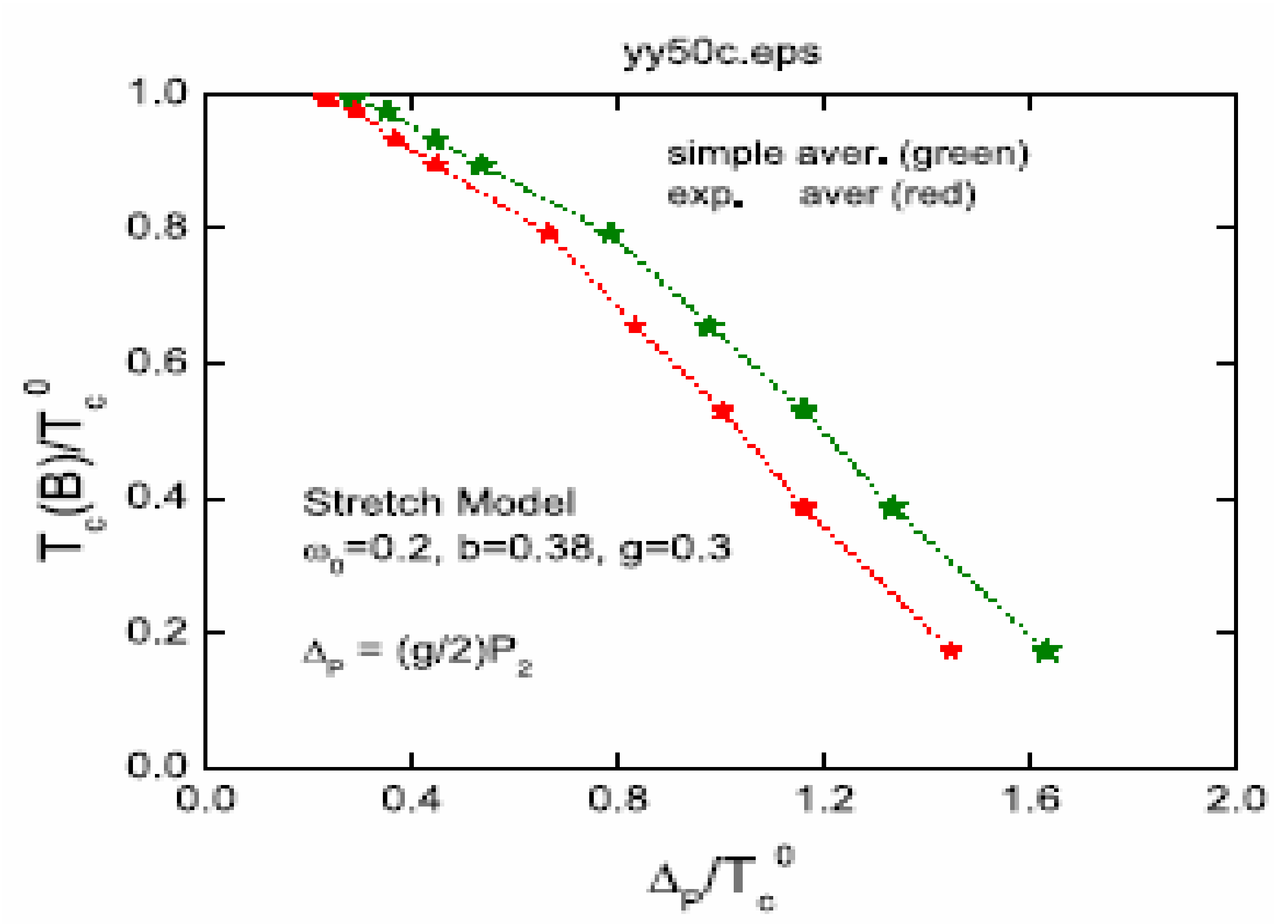
$$\Delta_P = \frac{1}{2D^\gamma(\gamma)}\delta_L\left(\frac{\Delta(0)}{\delta_L}\right)^\gamma$$

exceeds T_c **!**



Transition temperature v/s Pseudogap

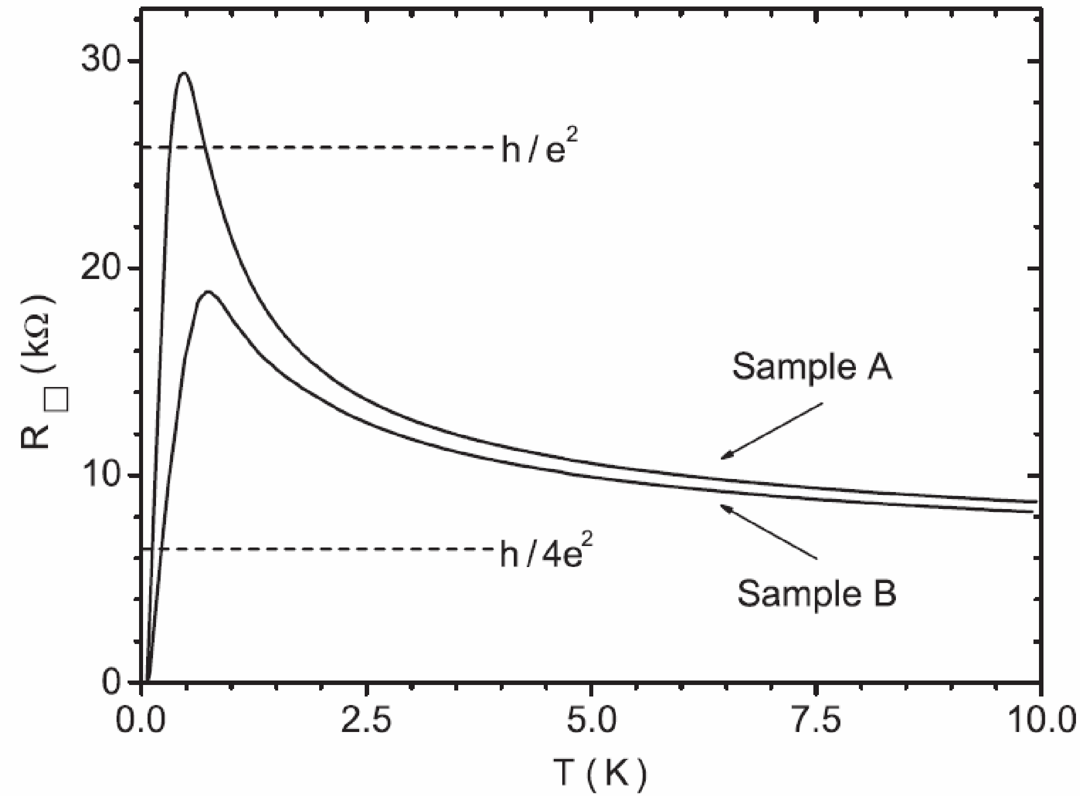
Virial expansion results:



Superconductivity with Pseudogap

The condition **Δ_P > Tc**

leads to “pseudogap phenomenology” :
a spectral gap opens above superconductive transition



Baturina et al , PRL 2007

With increase of disorder
Δ_P grows but **Tc** decreases

Major unsolved problems

- 1. Role of Coulomb enchancement near mobility edge ? (this effect was treated by Finkelstein for thin-film case)

- 2. How to include magnetic field into the "fractal" scheme ?

Conclusions

- Pairing of electrons on localized states leads to hard gap and Arrhenius resistivity
- Pairing on nearly-critical states produces fractal superconductivity with relatively high T_c but very small superconductive density
- Pseudogap behaviour is expected near S-I transition, with "insulating gap" exceeding T_c