

Thermodynamics (specific heat) of 1D Fermi systems

the role of renormalizations at intermediate scales

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Larkin conference, June 26, 2007

In collaboration with Dmitrii Maslov (U Florida)

Sasha Finkelstein, Monday's talk

The closely related work:

I. Aleiner and K. Efetov, Phys. Rev. B 74,
075102 (2006).

I. E. Dzyaloshinskii and A. I. Larkin
JETP 34, 422 (1972).

motivations

Specific heat in Fermi liquids, in dimensions $D > 1$

The leading term: $C(T)/T = \text{const}$ (**Fermi liquid theory**)

Subleading term is universal and non-analytic

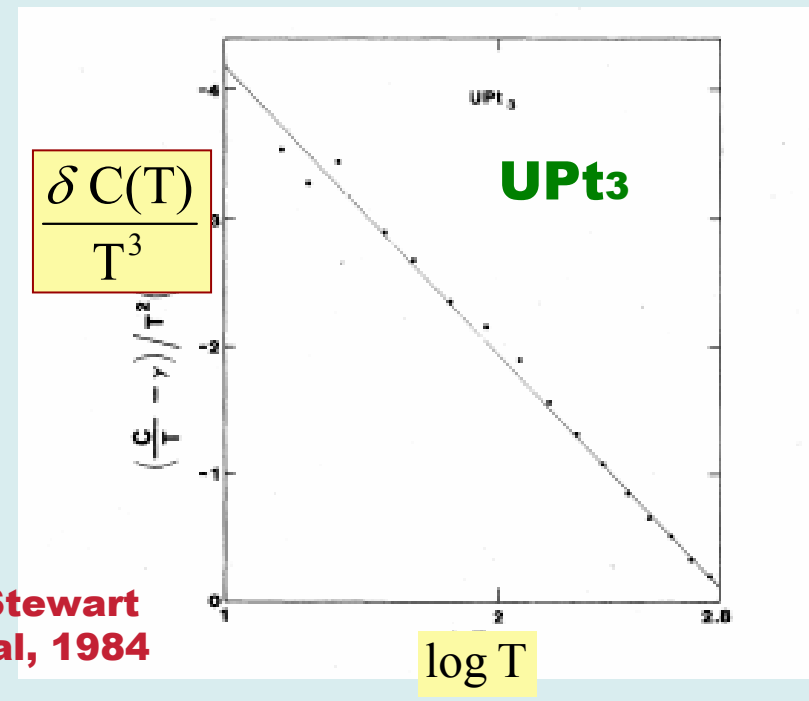
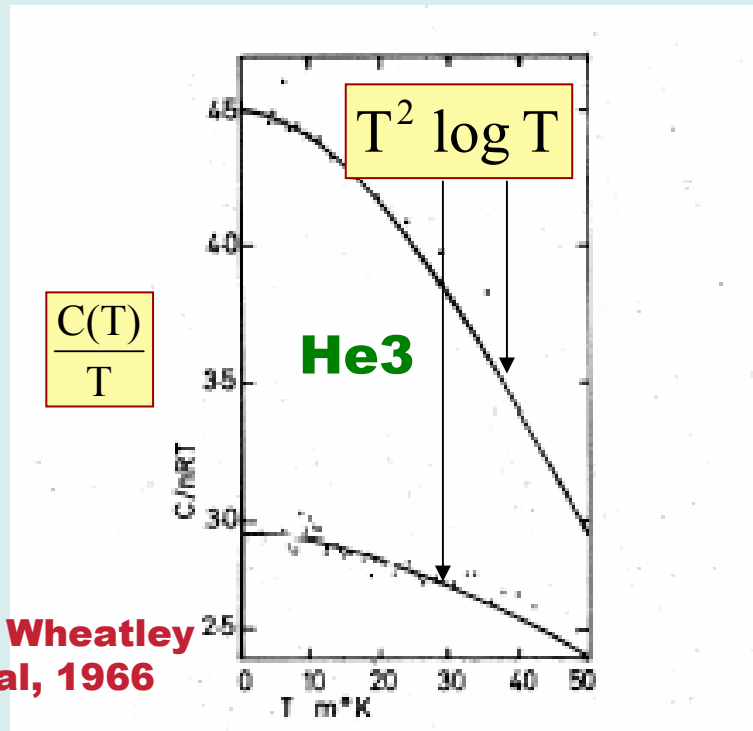
$$\frac{\delta C(T)}{T} \propto T^{D-1}$$

3D $C(T)/T \propto T^2 \log T$ **Eliashberg --- el-phonon**
Pethick & Carneiro --- el-el

3D Fermi-liquid

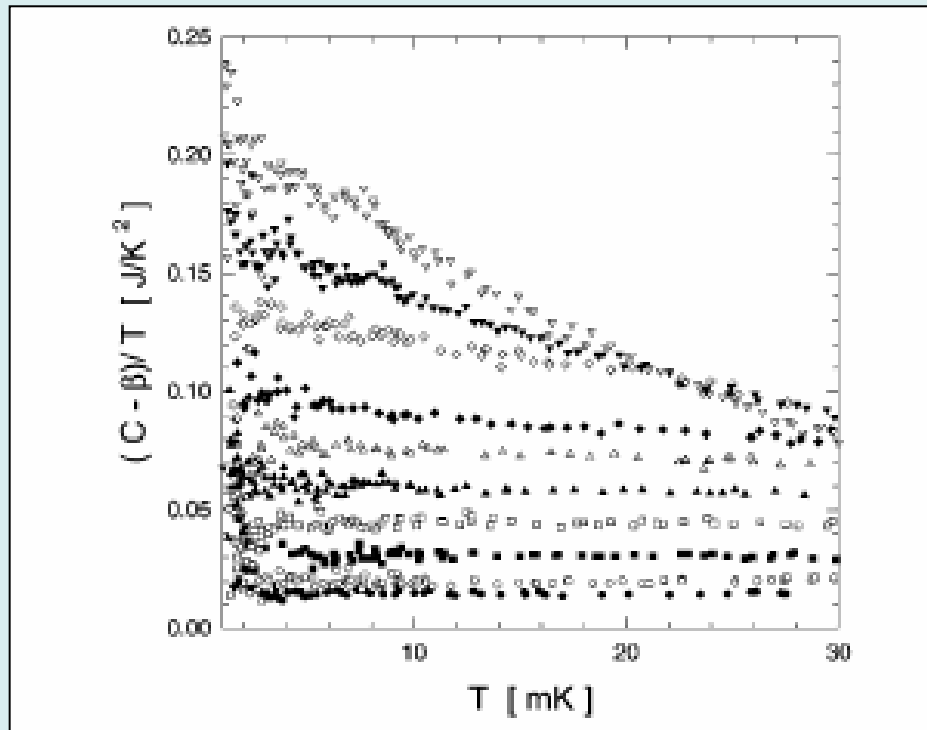
Specific heat:

$$\delta C(T)/T \propto T^2 \log T$$



2D Fermi-liquid

Monolayer of 3He



$$\frac{C(T)}{T} = \gamma - \Gamma \frac{T}{E_F}$$

A. Casey et al.
PRL 90, 115301 (2003)

Specific heat in Fermi liquids, in dimensions $D > 1$

The leading term: $C(T)/T = \text{const}$ (Fermi liquid theory)

Subleading term is universal and non-analytic

$$\frac{\delta C(T)}{T} = \frac{\delta C(T)}{T} \propto T^{D-1} \left(f(\pi) + 3 f_s^2(\pi) \right)$$

A.C., Maslov,
Glazman,
Gangadharaiah

$$f(\theta = \pi)$$



$$\Gamma(k, -k; k - k)$$

$$\Gamma(k, -k; -k, k)$$

charge **spin**
components of $f(\theta = \pi)$

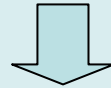
1D processes in $D > 1$ are responsible for non-analytic $\delta C(T)$

Another consequence of “one-dimensionality”

$$\frac{\delta C(T)}{T} = A_D T^{D-1} \left(f_c^2(\pi) + 3 f_s^2(\pi) \right)$$

$$f_c(\pi), f_s(\pi)$$

are combinations of interactions with zero total momentum



$$f_c(\pi), f_s(\pi) \propto 1/\log T$$

due to renormalization in the Cooper channel

**Aleiner & Efetov
Finkelstein & Shekhter
Efetov & Schwiete
A.C. & Maslov**

Now we move to actual 1D

1.

$$\frac{\delta C(T)}{T} = A_D T^{D-1} (f_c^2(\pi) + 3 f_s^2(\pi)), \quad T^{D-1} \Rightarrow \text{const}$$

2.

In 1D, charge backscattering amplitude remains finite

Spin backscattering amplitude scales as

$$f_s(T) = \frac{f_s}{1 - 2 f_s |\log T|}, \quad f_s < 0$$



Bychkov, Gor'kov & Dzyaloshinskii
Dzyaloshinskii & Larkin

One might expect that

• **charge component**

$$\frac{C_c(T)}{T} = \text{const}$$

• **spin component**

$$\frac{C_s(T)}{T} \sim \left(\frac{f_s}{1 - 2 f_s |\log T|} \right)^2$$

By this logics, $T \log T$ should appear at the third order

The actual result is different:

- **T log T only appears at the fourth order**

- **the spin component**

$$\frac{C_s(T)}{T} \sim \left(\frac{f_s}{1 - 2 f_s |\log T|} \right)^3$$

Aleiner & Efetov -- “bosonization”
Wiegmann & Tsvelik – Kondo model
Luk’yanov -- XXZ S=1/2 model

What’s special about backscattering in 1D?

We did “brute force” perturbative calculations for 1D fermions with short-range interaction.

g-ology in 1D:

Giamarchi

g₄ – forward scattering

$$\Gamma(k, k; k, k)$$

g₂ – charge part of backscattering

$$\Gamma(k, -k; k - k)$$

g₁ – 2p_F scattering (spin part of backscattering)

$$\Gamma(k, -k; -k, k)$$

The spin contribution to the specific heat comes from 2p_F scattering (g₁ processes)

I will only be talking about 2k_F processes

In essence, the issue is the following:

In perturbation theory in 1D, $C(T)$ is expanded in powers of g_1 , beginning from the first order.

$$\frac{C(T)}{T} = \frac{2\pi}{v_F} (a_1 g_1 + a_2 g_1^2 + a_3 g_1^3 + a_4 g_1^4 + \dots)$$

$$a_i = a_i (\log T)$$

$$g_1(E) = \frac{g_1}{1 + 2 g_1 |\log E|}$$

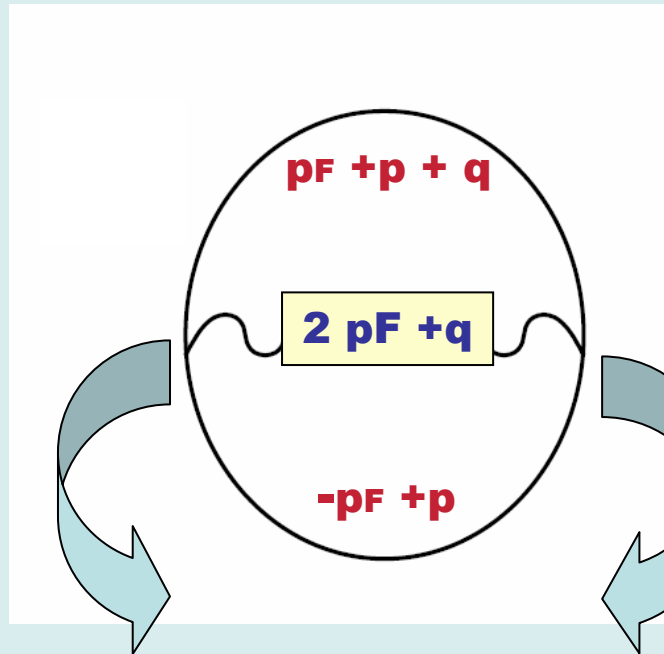
The coupling g_1 in $C(T)$ can be

at the scale of T , $E \sim T$, g_1 is the RG- running coupling

If, at a given order, g_1 in $C(T)$ is the coupling at the scale of T , one should obtain $\log T$ at the next order.

What it means that g_1 is at a particular scale?

$$\frac{C(T)}{T} = - \frac{\partial^2 \Xi}{\partial T^2}$$



**1st order
2pF contribution**

$$g_1 = \Gamma(p_F + p + q, -p_F + p; -p_F + p, p_F + p + q)$$

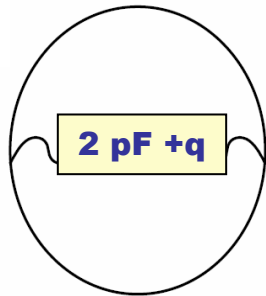


If p , or $q = O(\text{cutoff})$

$g_1 \approx \Gamma(p_F, -\frac{g_1}{T} - p_F, p_F)$ **is the running vertex at the scale $O(T)$**
vertex at the scale of the cutoff

Generally, contributions from momenta of the order of the cutoff scale as powers of the cutoff and can be easily separated from universal contributions from momenta $O(T)$.

However, in 1D, we have “anomalies” – the contributions which are cutoff independent, but can be obtained as low-energy contributions **OR as high-energy contributions.**



$$\Xi_1(2p_F) = -g_1 T \sum_{\Omega} \int_{-\Lambda_b}^{\Lambda_b} dq \Pi_{2p_F}(q, \Omega)$$

$$\Pi_{2k_F}(q, \Omega) = \frac{1}{2\pi} \left(\log \frac{\Omega^2 + q^2}{4\Lambda_f^2} - 8 \int_0^{\infty} n_F(x) x dx \left(\frac{1}{(q - i\Omega)^2 - 4x^2} + \frac{1}{(q + i\Omega)^2 - 4x^2} \right) \right)$$

$$T \sum_{\Omega} \int_{-\Lambda_b}^{\Lambda_b} dq \Pi_{2p_F}(q, \Omega) = \frac{\pi}{3} T^2$$

Take a symmetrized combination: the answer

$$\Xi_1(T) = \frac{\pi}{3} T^2 \quad \text{comes solely from } q, \Omega = O(\Lambda_b)$$

The model:

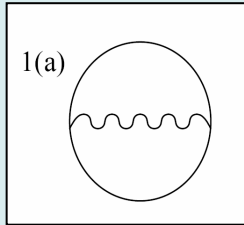
Fermions with linearized dispersion $\varepsilon_p = v_F (p - p_F)$
up to a cutoff at $|p - p_F| = \Lambda_f$

2pF interaction $U(2p_F + q)$ **extending up to**
a cutoff at $|q| = \Lambda_b$

Assume $\Lambda_b \ll \Lambda_f$, **the same condition**
as in conventional bosonization.

If we set $\Lambda_b \sim T$, **the contribution to** $\Xi_1 = \pi T^2 / 3$ **from “high-**
energy cutoff at $q \sim \Lambda_b$ **becomes the contribution from** $q \sim T$

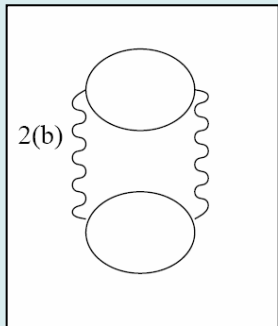
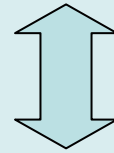
The results:



1. First order

$$\Xi_1 = -\frac{\pi}{3} T^2 g_1$$

2. Second order



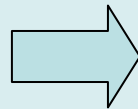
$$\Xi_2 = \frac{\pi}{3} T^2 g_1 \left(2 g_1 \log \frac{\Lambda_f}{\Lambda_b} \right) - \frac{\pi}{3} T^2 g_1^2$$

1st order renormalization of
the backscattering vertex

this term is
 $T^2 f_s^2(\pi)$

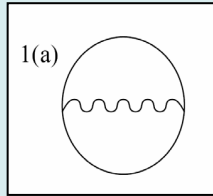
RG

$$g_1(E) = \frac{g_1}{1 + 2 g_1 |\log E|}$$



**the coupling g_1 is at the intermediate,
but still a finite energy scale $E = \Lambda_f/\Lambda_b$**

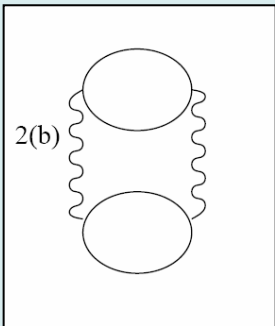
The results:



1. First order

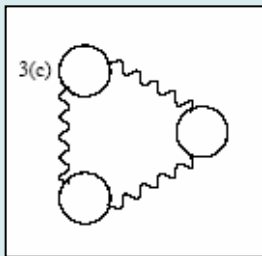
$$\Xi_1 = -\frac{\pi}{3} g_1 T^2$$

2. Second order



$$\Xi_2 = \frac{\pi}{3} T^2 g_1 \left(2 g_1 \log \frac{\Lambda_f}{\Lambda_b} \right) - \frac{\pi}{3} T^2 g_1^2$$

3. Third order



$$\Xi_2 = -\frac{\pi}{3} T^2 g_1 \left(4 g_1^2 \log^2 \frac{\Lambda_f}{\Lambda_b} \right) + \frac{\pi}{3} T^2 g_1^2 \left(4 g_1 \log \frac{\Lambda_f}{\Lambda_b} \right)$$

2nd order renormalization of
the backscattering vertex

1st order renormalization of
the backscattering vertex

g_1 in $C(T)$ is the backscattering at the scale of the cutoff set by the interaction

Is there a coupling at the scale of T?

Third order, but more carefully



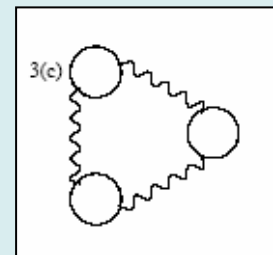
$$\Xi_2 = -\frac{\pi}{3} T^2 g_1 \left(4 g_1^2 \log^2 \frac{\Lambda_f}{\Lambda_b} \right) + \frac{\pi}{3} T^2 g_1^2 \left(4 g_1 \log \frac{\Lambda_f}{\Lambda_b} \right) + 0^* (T^2 g_1^3)$$

$$0^* (T^2 g_1^3) = \underbrace{\pi T^2 g_1^3}_{\text{same as at first and second order } g_1(\Lambda)} - \underbrace{\pi T^2 g_1^3}_{\text{comes from } q, \Omega \rightarrow 0}$$

same as at first and second order $g_1(\Lambda)$

comes from $q, \Omega \rightarrow 0$

$$-\pi T^2 g_1^3 \Leftarrow \pi T g_1^3 T \Sigma_{\Omega_1} T \Sigma_{\Omega_2} (1 - \delta_{\Omega_1,0} \delta_{\Omega_2,0})$$



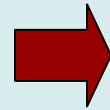
The same type of frequency sums gives rise to the non-analyticity in the spin susceptibility

Fourth and higher orders:

$$\bar{E} \Rightarrow -\pi T^2 (g_1^3(T) - g_1^3(\Lambda))$$



$$g_1(T) = \frac{g_1}{1 + 2g_1 |\log \Lambda/T|}$$



$$\frac{C(T)}{T} \sim g_1^4 \log T + \dots$$

Final result (including g4, g2)

$$\frac{C(T)}{T} = \underbrace{\frac{\pi}{3} \left(\frac{1}{v_\rho} + \frac{1}{v_\sigma} \right)}_{\text{bosonization result}} + \underbrace{\frac{\pi}{3} g_1^2}_{\text{bosonization result}} + \underbrace{2\pi \frac{g_1^3}{(1 + 2g_1 \log \Lambda/T)^3}}_{\text{bosonization result}}$$

$$g_1 = g_1(\Lambda_b)$$

bosonization
result, v_ν, v_σ
are charge and
spin velocities

$$v_\rho = \left((1 + 2g_4 - g_1)^2 - (2g_2 - g_1)^2 \right)^{1/2}$$

$$v_\sigma = \left((1 - g_1)^2 - g_1^2 \right)^{1/2}$$

h the
(T)

Efetov

Conclusions:

The appearance of the backscattering coupling at some order in $C(T)$ with apparently universal, cutoff-independent prefactor does NOT imply that this coupling is the running one at the scale of T .

The running coupling at the scale of T does appear in the specific heat at the third order.

$$\frac{C(T)}{T} = \frac{\pi}{3} \left(\frac{1}{v_\rho} + \frac{1}{v_\sigma} \right) + \frac{\pi}{3} g_1^2 + 2\pi \frac{g_1^3}{(1 + 2g_1 \log \Lambda/T)^3}$$

**N
E
W**

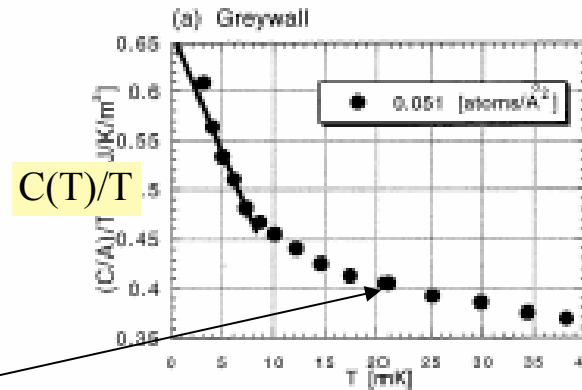
g_1 is renormalized down to the scale set by the interaction

an additional constant backscattering contribution to $C(T)$

THANK YOU!

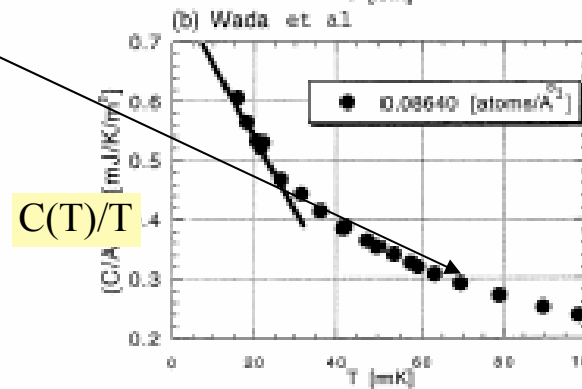
Monolayer of 3He

Greywall



T

Wada



**Experiments
in mid-80th**

$$f_c^2(\pi) + 3f_s^2(\pi) \sim 15$$