

“What are topological excitations? ... Think about local effects!”

- A.I. Larkin, in memoirs of V.L. Pokrovskii

Solitons in microscopic electronic processes in correlated systems Recent experiments in light of Larkin inspirations

1960' s : *Larkin – Ovchinnikov ; Fulde – Ferrell*

Inhomogeneous state of spin – polarized superconductors

Larkin – Khmel'nitskii ; Nagaev

Inhomogeneous state of doped ordered medias –
Ferroelectric and magnetic conductors

1970's *Larkin – Dzyaloshinskii ; Luther – Emery*

Origin of the Mott AFM state in quasi 1D conductors

1980's *Larkin – Ovchinnikov, Korshunov ;
following Caldeira – Leggett*

Instantons in dissipative quantum multi-state systems

Solitons in quasi one dimensional conductors *and beyond*

- I. Ferroelectric Mott-Hubbard phase and charge ordering in organic conductors – a firework of solitons
- II. Solitons in dynamics and in the ground state of overlap tunnelling junctions of incommensurate Charge Density Waves
- III. Role of topological excitations in general strongly correlated systems

Solitons/instantons in electronic properties:

Born in theories of late 70's, Found in experiments of early 80's.

Why in 2000's ?

New conducting polymers,

New events in organic conductors,

New accesses to Charge Density Waves,

New interests in strongly correlated systems as semiconductors

Sources for this talk :

S.B. collaborations with joint experimental groups of

Grenoble (Monceau) and Moscow (F.Nad, Yu.Latyshev, et al),

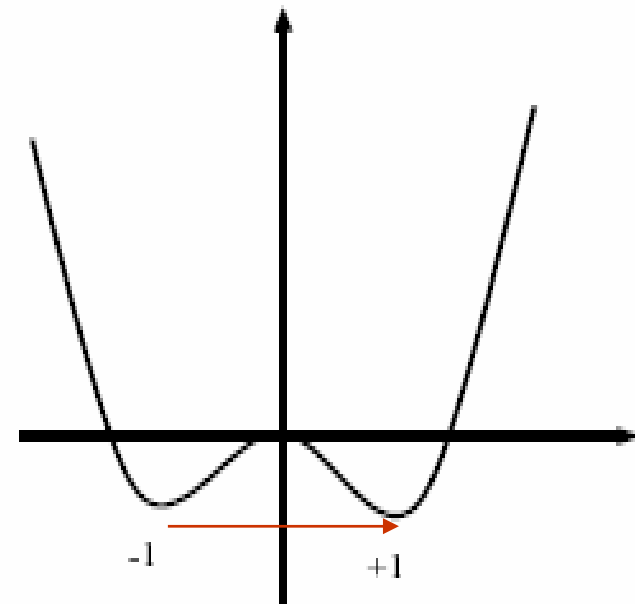
Earlier theory work (S.B., N.Kirova, S.Matveenko, et al, etc.)

Solitons in general : propagating isolated profiles –
from the tsunami wave (Russel 1834) to
electron density droplets (Abanov & Wiegmann - today)

Solitons in our perspective of electronic systems :
Nonlinear self-localized excitations on top of a ground state with a
spontaneously broken symmetry: superconductivity, CDW, SDW, AFM
Solitons carry a charge or a spin – separately, even in fractions.
They bring associated spectral features (e.g. mid-gap states)
Instantons : related transient processes for creations of solitons or
their pairs, for conversion of electrons or excitons into solitons

Symmetry breaking :
Degenerate equivalent ground states.
Soliton = kink between them

Figure : energy as a function of configuration.
two-fold degeneracy or
cross-section of the axial-symmetry shape

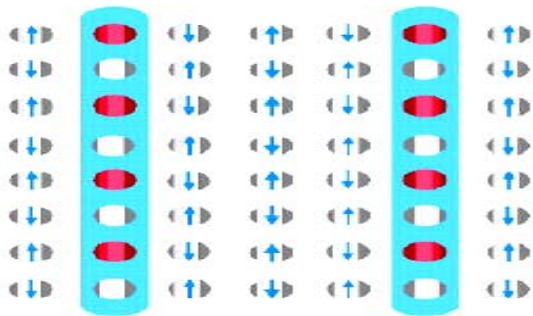


LOFF rout: from stripes to solitons

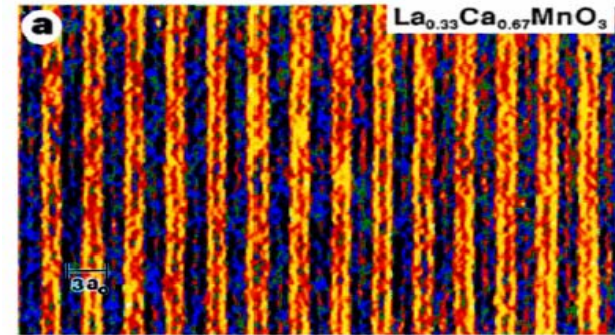
$1D \rightarrow quasi\ 1D \rightarrow 2D, 3D$ route to dopping of AFM insulator.
Aggregation of holes (extracted electrons) into stripes.

Left: scheme derived from neutron scattering experiments.

Right: *direct visualization via electron diffraction microscope.*



J.Orenstein et al Science 288, 468 (2000)



S.Mori et al Nature 392, 473 (1998)

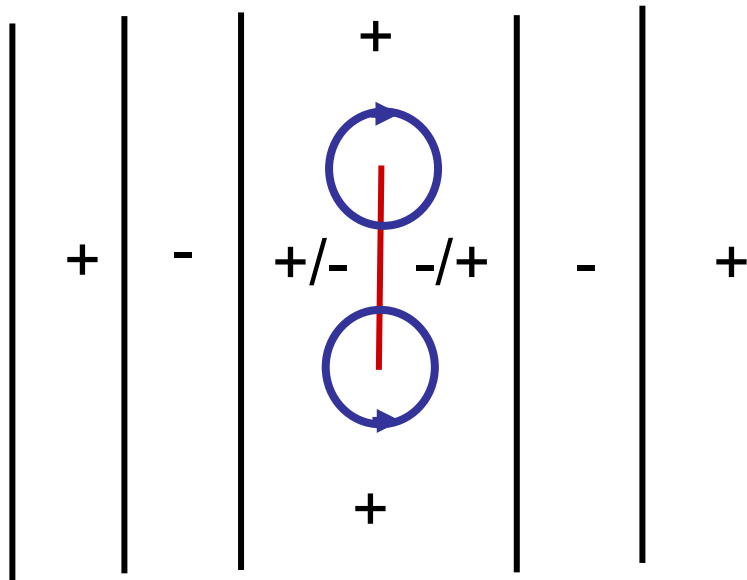
Equivalence for spin-gap cases:

Fulde-Ferrell-Larkin-Ovchinnikov FFLO phase in superconductors

Solitonic lattices in CDWs above the magnetic breakdown

Solitonic lattices in spin-Peierls GeCuO in HMF - Grenoble

Kink-roton complexes as nucleuses of melted macro structures:
 FFLO phase for superconductors or strips for doped AFMs.



A defect embedded into the regular stripe structure (black lines).
 +/- are the alternating signs of the order parameter amplitude.

Termination points of a finite segment (red color) of the zero line must be encircled by semi-vortices of the π rotation (blue circles) to resolve the signs conflict.

The minimal segment corresponds to the spin carrying kink.

Major and unifying observation :

combination of a discrete and continuous symmetries

Solitons are stable energetically but not topologically

Special significance: allowance for a direct transformation of
one electron into one soliton.

(Only $2 \rightarrow 2$ are allowed for kinks in discrete symmetries)

Complex Order Parameter

$O = A \exp[i\varphi]$; **A** - amplitude , **φ** - phase

Ground State with an odd number of particles:

In 1D - *Amplitude Soliton AS* **$O(x=-\infty) \leftrightarrow -O(x=\infty)$**

via **$A \leftrightarrow -A$** at arbitrary $\varphi = \text{cnst}$

Favorable in energy in comparison with an electron, **but:**

Prohibited to be created dynamically even in 1D

Prohibited to exist even stationary at $D > 1$

RESOLUTION – Combined Symmetry :

$A \leftrightarrow -A$ combined with **$\varphi \rightarrow \varphi + \pi$** – **semi-vortex of phase rotation**
compensates for the amplitude sign change

MIXED DISCRETE AND CONTINUOUS SYMMETRIES.

SPIN-GAP cases: Incommensurate CDW or Superconductor

$$H_{1D} \sim (\partial\theta)^2 - V \cos(2\theta) + (\partial\varphi)^2$$

V - from the backward exchange scattering of electrons

In **1D** : Spinon as a soliton $\theta \rightarrow \theta + \pi$ hence **$s=1/2$**

+ Gapless charge sound in φ .

$$\text{CDW order parameter} \sim \psi_{+\uparrow}^\dagger \psi_{-\uparrow} + \psi_{+\downarrow}^\dagger \psi_{-\downarrow} \sim \exp[i\varphi] \cos\theta$$

- Its amplitude **$\cos\theta$** changes the sign along the allowed π soliton

At higher D : allowed mixed configuration

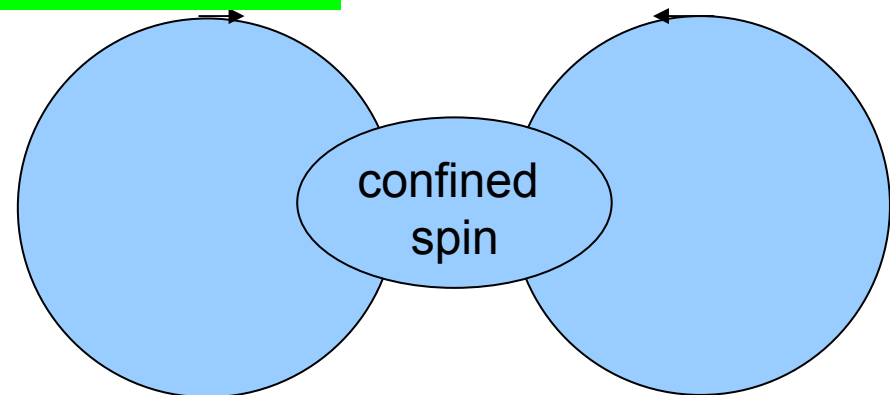
$$\theta \rightarrow \theta + \pi, \quad s=1/2$$

↑ spin soliton ↑

$$\varphi \rightarrow \varphi + \pi, \quad e=1$$

↑ charged wings ↑

Spinon as a soliton +
semi-integer dislocation loop =
 π - vortex of $\varphi \equiv$ confined spin +
semi dislocation loop



Singlet Superconductivity:

$$D=1 \rightarrow D>1$$

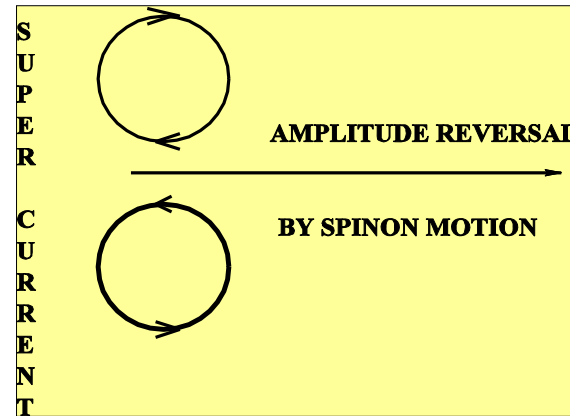
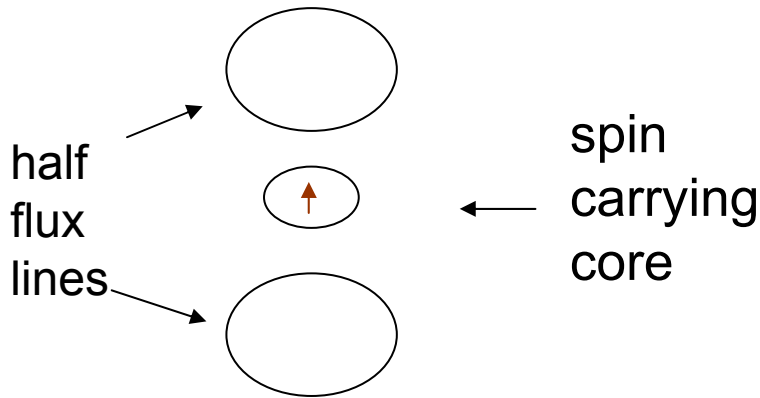
$$\eta_{\text{SC}} \sim \psi_{+\uparrow} \psi_{-\downarrow} + \psi_{+\downarrow} \psi_{-\uparrow} \sim \exp[i\chi] \cos\theta$$

$$\theta \rightarrow \theta + \pi \quad s=1/2$$

$$\chi \rightarrow \chi + \pi$$

↑ spin soliton ↑

↑ wings of supercurrents ↑



Quasi 1d view : spinon as a π - Josephson junction in the superconducting wire (applications: Yakovenko et al).

2D view : pair of π - vortices shares the common core bearing unpaired spin.

3D view : half-flux vortex stabilized by the confined spin.

Best view: nucleus of melted FFLO phase in spin-polarized SC

Half filled band with repulsion.
SDW rout to the doped Mott-Hubbard insulator.

$$H_{1D} \sim (\partial\varphi)^2 - U \cos(2\varphi) + (\partial\theta)^2$$

U - Umklapp amplitude

(*Dzyaloshinskii & Larkin ; Luther & Emery*).

φ - chiral phase of charge displacements

θ - chiral phase of spin rotations.

Degeneracy of the ground state:

$\varphi \rightarrow \varphi + \pi =$ translation by one site

Excitations in 1D :

holon as a π soliton in φ , spin sound in θ

Higher D : A hole in the AFM environment.

Staggered magnetization \equiv AFM=SDW order parameter:

$$O_{SDW} \sim \cos(\varphi) \exp\{\pm i(Qx + \theta)\}$$

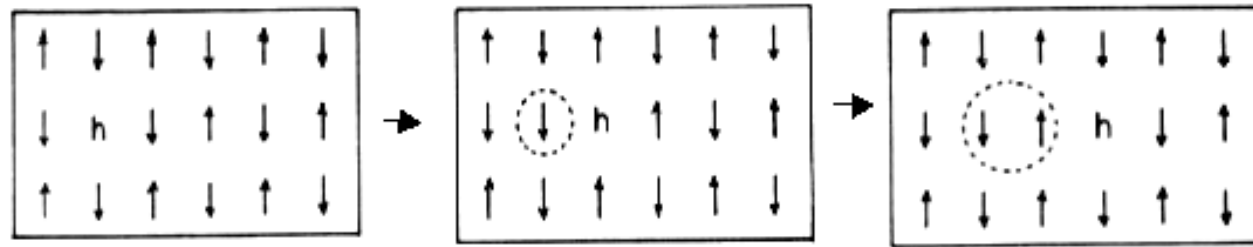
To survive in $D > 1$:

The π soliton in φ $\cos \varphi \rightarrow -\cos \varphi$

enforces a π rotation in θ to preserve O_{SDW}

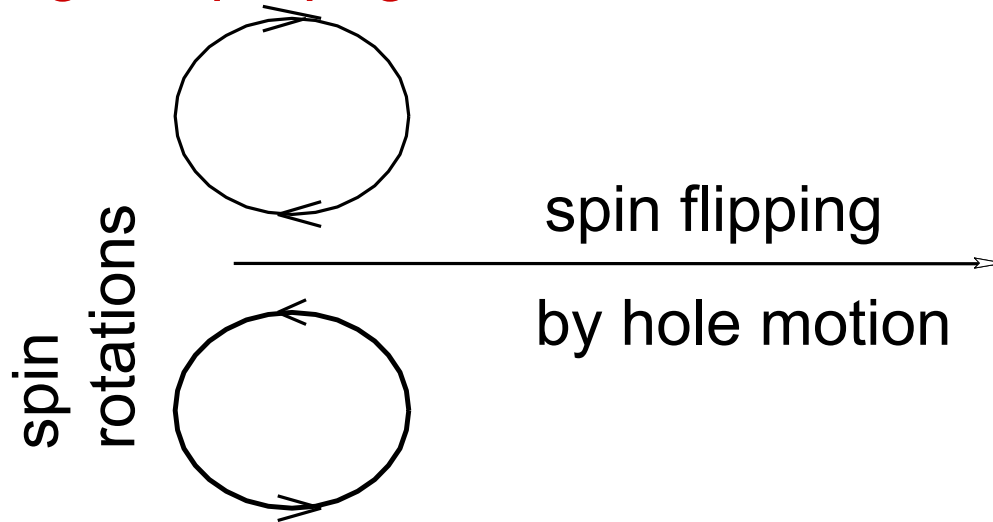
Propagating hole as an amplitude soliton.

Its motion permutes AFM sublattices \uparrow, \downarrow creating a string of the reversed order parameter: staggered magnetization. It blocks the direct propagation.



*Nagaev et al ,
Brinkman and Rice*

Adding the semi-vorticity to the string end heals the permutation thus allowing for propagation of the combined particle.



Alternative view:

Nucleus of the stripe phase or the minimal element of its melt.

Solitons and their arrays in incommensurate CDWs

Yu.Latyshev, P.Monceau, S. Brazovskii

Observation of Charge Density Wave Solitons in Overlapping Tunnel Junctions Phys. Rev. Lett., **95**, 266402 (2005)

Subgap collective tunnelling and its staircase structure in charge density waves Phys. Rev. Lett., **96**, 116402 (2006).

More on theory side :

S.B. and Serguei Matveenko, cond-mats/2000'

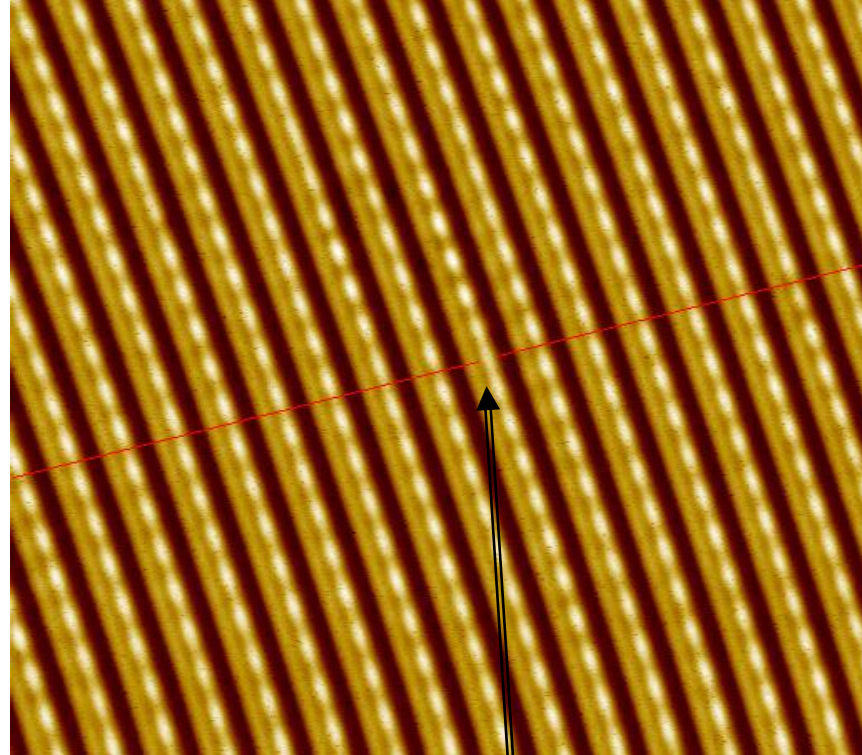
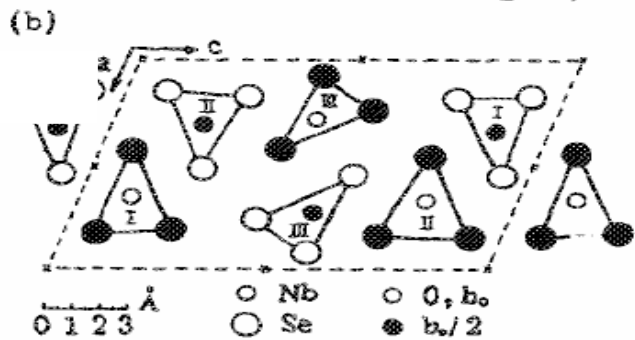
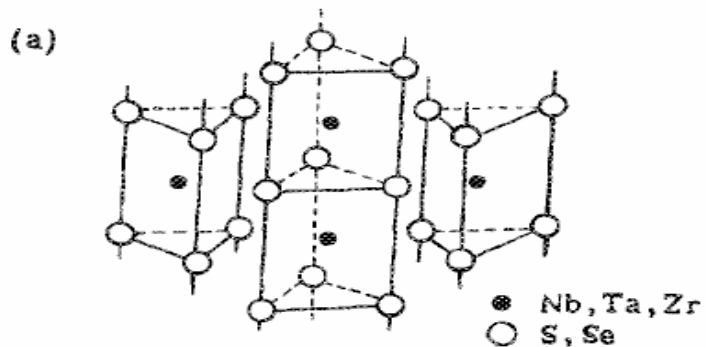
Instanton theory of the pseudogaps and of subgap transitions :
Applications to PES, ARPES, optics, tunneling.

Incommensurate Charge Density Wave – ICDW $\sim \cos(Qx + \varphi)$

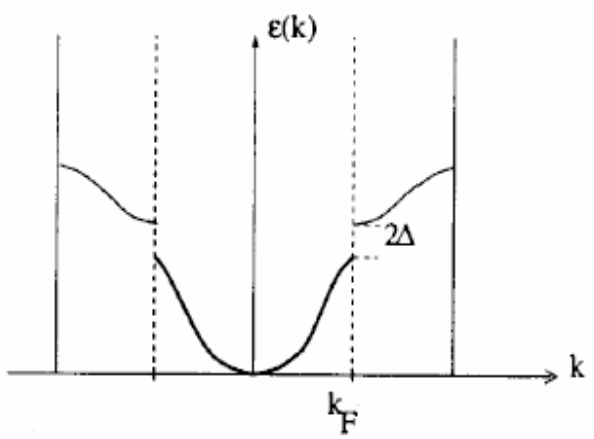
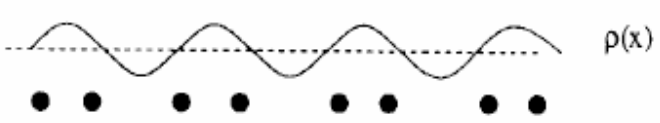
Minimal view : uniaxial crystal of singlet electronic pairs,
defined on chains of the parent crystal.

Incommensurability allows for arbitrary ICDW displacements,
hence the complex order parameter $A \exp(i \varphi)$

- isomorphisme to superconductivity (*Efetov and Larkin*)



2e hole of the CDW crystal NbSe₃
 = 2π soliton = vortex pair =
 result of the 2π phase slip
 STM image, C. Brun and Z.Z. Wang



CDW period $\lambda = \pi / k_f$

density $\sim \cos(2k_f x + \varphi)$

Gap opened at the Fermi level

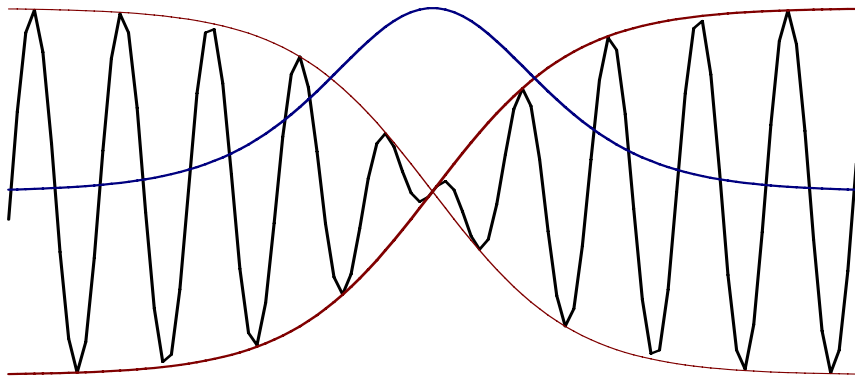
What should happen if the singlet pair is broken into spin $\frac{1}{2}$ components ?

This creature appears in tunneling (*S.B.*, 3 decades anniversary):

Amplitude soliton with **energy** $\approx 2/3\Delta$,

total charge 0, spin $\frac{1}{2}$

This is the CDW realization of the SPINON



Oscillating electronic density,
Overlap soliton $A(x)$,
Midgap state = spin distribution

Solitons as a result of electrons' selftrapping – next slide :

Microscopics of electrons conversion in ICDW:

Incommensurate CDW : $A \cos(Qx + \varphi)$ $Q = 2K_f$

Order parameter : $\Delta \sim A \exp(i\varphi)$

Electronic states $\Psi = \Psi_+ \exp(iK_f x + i\varphi/2) + \Psi_- \exp(-iK_f x - i\varphi/2)$

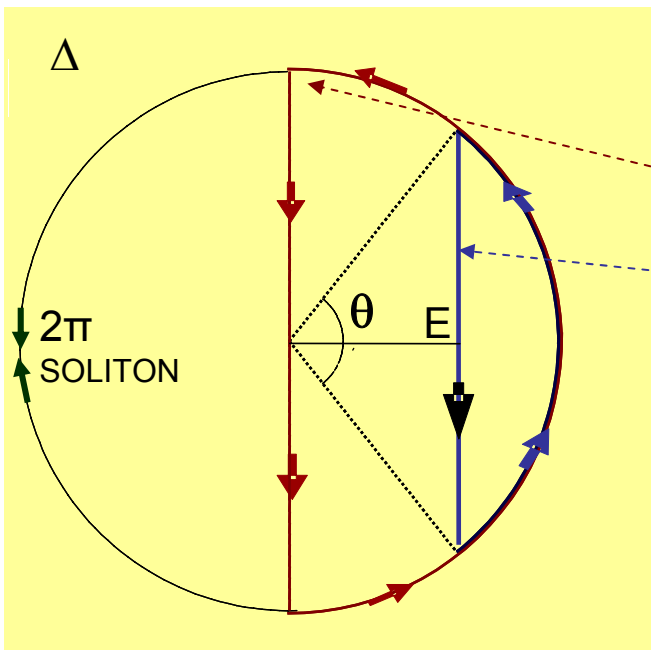
$$\begin{vmatrix} k - E & \Delta^* \\ \Delta & -k - E \end{vmatrix} \begin{pmatrix} \Psi_+ \\ \Psi_- \end{pmatrix} = 0, \quad k = K - K_f = -i\partial_x$$

Peierls-Frohlich, chiral Gross-Neveu models.

Spectra are related to the nonlinear Schroedinger equation for Δ :
Fateev, Novikov, Its, Krichever; Matveenko and S.B.

In equilibrium : $\Delta = \Delta_0 = \mathbf{const}$, $E = \pm(\Delta_0^2 + k^2)^{1/2}$

Major interest: spectral flow between the two branches of allowed states $E > \Delta_0$ and $E < -\Delta_0$ and the related conversion of added particles to the extended ground state



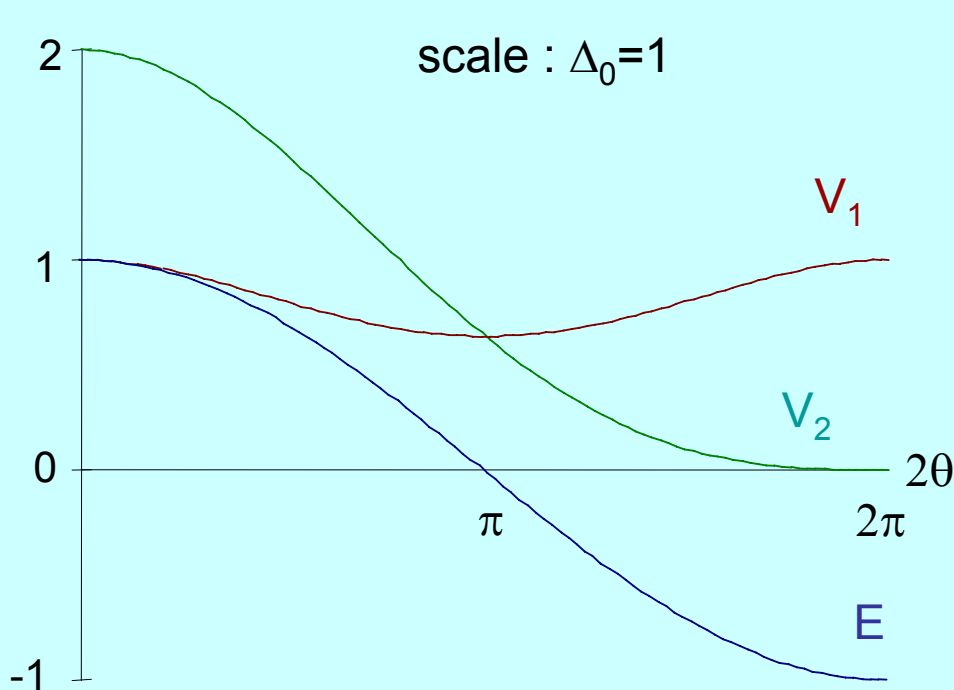
Soliton trajectories in the complex plane of the order parameter.

Red line: stable amplitude soliton.

Blue line: intermediate chordus soliton within chiral angle θ (black radial lines).

The value $\theta=100^\circ$ is chosen which corresponds to the optimal configuration for the interchain tunnelling

S. Matveenko and S.B.



Selftrapping branches $V_n(\theta)$ for chordus solitons with fillings $n=1$ and $n=2$, Energy $E(\theta)$ of localized split-off state - Spectral flow between gap edges $\Delta_0 \rightarrow -\Delta_0$

No barrier for selftrapping in 1D !

Incommensurate CDWs in quasi 1D conductors.

Subgap transitions due to non-adiabatic quantum fluctuations;

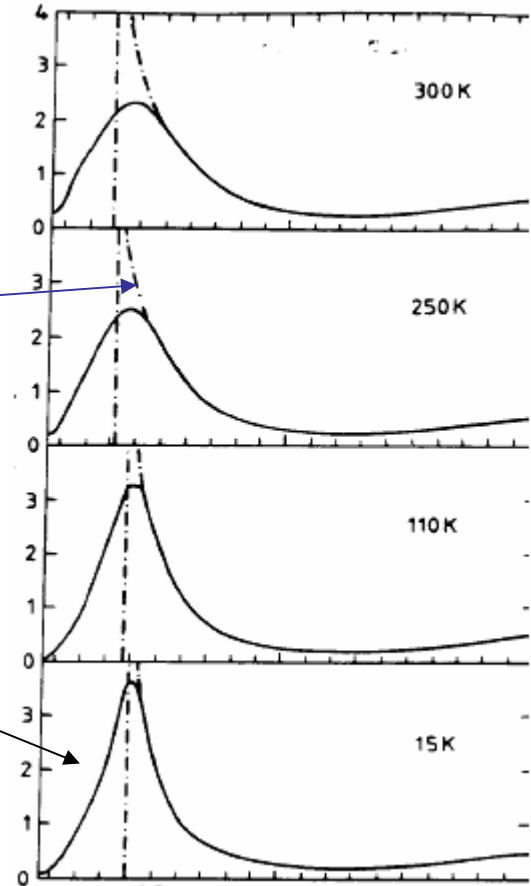
Related pseudogap (*cf. cuprates*).

Already a long standing problem in optics :

Figure – DiGiorgi group, ETH
Optical absorption in Blue Bronze

Expected edge singularity at 2Δ

Subgap tail persistent at low T.
Quantum processes -
presence of excitations below 2Δ



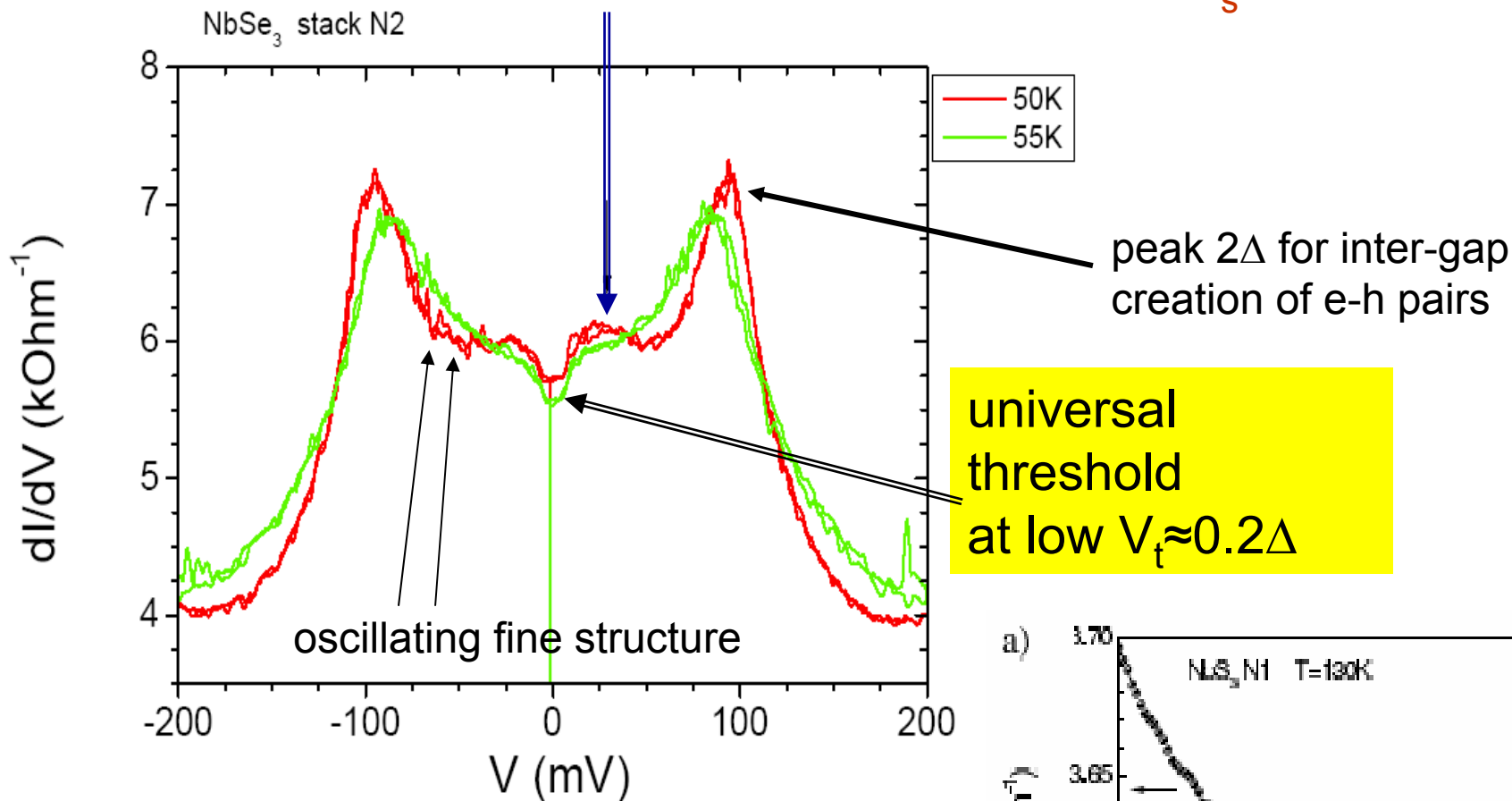
Recovering a full and rich picture – New access by experiments of tunneling, particularly in overlap mesa-junctions

(Latyshev, Monceau, Sinchenko, S.B. – IRE&MIFI&ITP, Grenoble, Orsay)

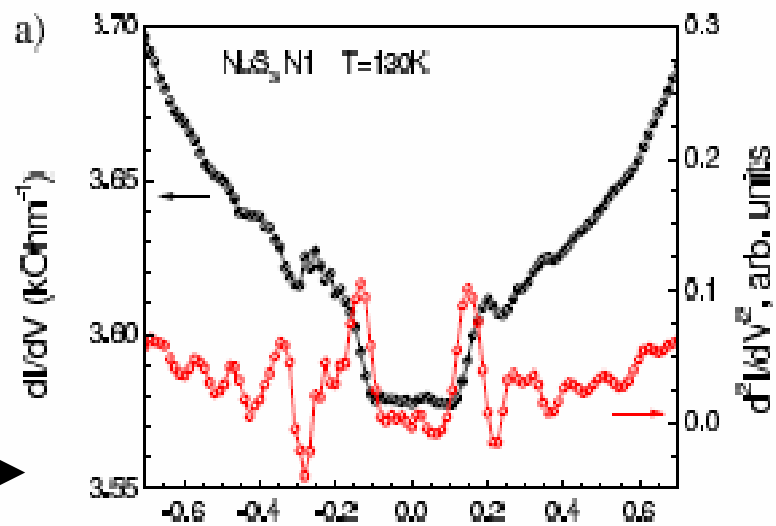
Direct observation of solitons and their arrays in tunneling on NbSe₃

All feature scale with $\Delta(T)$!

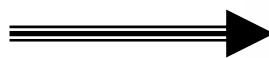
creation of solitons at $2\Delta/\pi : E_s = 2\Delta/\pi$!



universal threshold at low $V_t \approx 0.2\Delta$



fine structure is **not a noise** !
its interpretation :
sequential entering of dislocation lines
into the junction area.

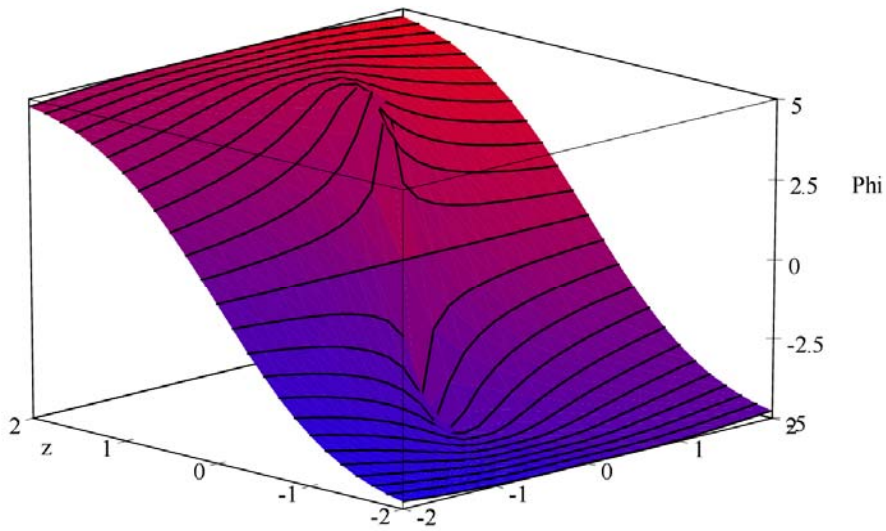


Fine structure within the magnified threshold region.
Conductance dI/dV and its derivative d^2I/dV^2
Voltage V is normalized to the CDW gap.

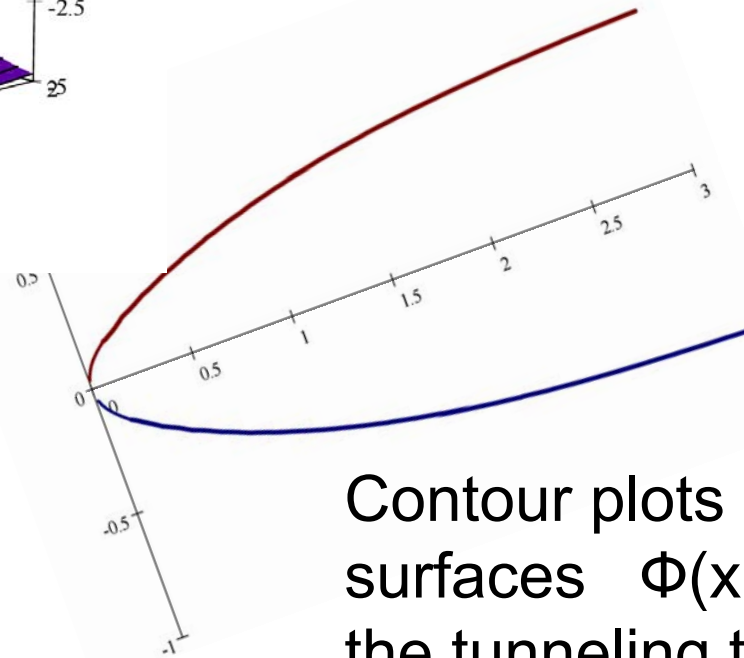
Comparison of d^2I/dV^2 for
two voltage polarities for both CDWs,
at $T=130\text{K}$ and 50K ;

the positive polarity at $T=120\text{K}$

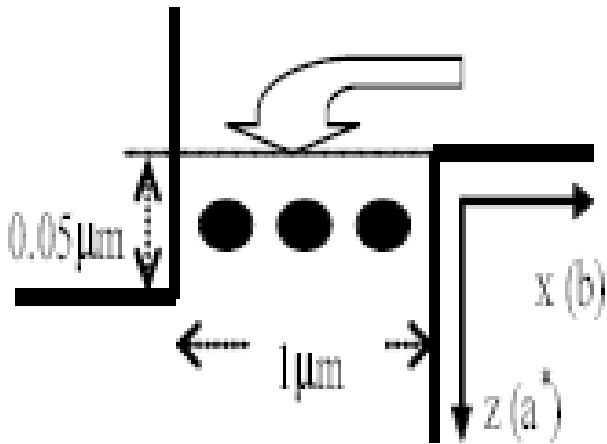
Peaks interpretation :
sequential entering of dislocation lines into the junction area.
Critical voltage - DL entry energy, like H_{c1} in superconductors.
(*S.B.-Matveenko 1989*)



Potential distribution in a DL vicinity. Notice concentration of the potential $\Phi(x,z)$ drop, facilitating the tunneling.



Contour plots $\pm z(x)$ for surfaces $\Phi(x,z) \pm \Delta$ where the tunneling takes place.



Junction scheme with crosssections of dislocations

Pair of $\pm 2\pi$ solitons is created by tunneling near the dislocation core, Interpretation: excitation of the dislocation line as a quantum string.

Phase mode action : (u - phase velocity)

$$S_{snd} = \frac{v_f}{4\pi} \int \int dx dt \left((\partial_t \varphi / u)^2 + (\partial_x \varphi)^2 \right) \quad \varphi(t, \pm 0) = \mp \theta(t)$$

The discontinuity is enforced by the Chordus soliton forming around $x_s=0$

Integrate out $\varphi(x,t)$ from $\exp(-S_{snd}[\varphi, \theta])$ - arrive for $\theta(t)$
at the typical action for the problem of quantum dissipation

$$S \sim \sum |\omega| |\theta_\omega|^2 \quad \dot{\theta} = \partial_t \theta$$

$$S_{snd} \{\theta\} \approx -\frac{v_F/u}{2\pi^2} \int \int dt_{1,2} \dot{\theta}(t_1) \ln|(t_1 - t_2)| \dot{\theta}(t_2)$$

$$S_{snd} \approx (v_F/4u) \ln(uT/\xi_0)$$

AS edge $\Omega \geq W_s$ - the power law $I(\Omega) \propto \left(\frac{\Omega - W_s}{W_s} \right)^\beta$, $\beta = \frac{v_F}{4u}$

Summary for tunneling:

Specifics of strongly correlated electronic systems
inorganic CDW, organic semiconductors,
conjugated polymers, conducting oxides, etc...

Electronic processes, in junctions at least, are governed by solitons or more complex nonlinear configurations.

As proved by presented experiments, they can lead to:

- Conversion of a single electron into a spin solitons
- Conversion of electrons pair into the 2π phase slip
- Pair creation of solitons (tunneling and optics)
- Arrays of solitons aggregates
- dislocation lines, walls of discommensurations –
reconstruct the junction state and provide
self-assembled micro-channels for tunneling;

1D Mott-Hubbard state. 1 electron per site i.e. the half filled band.

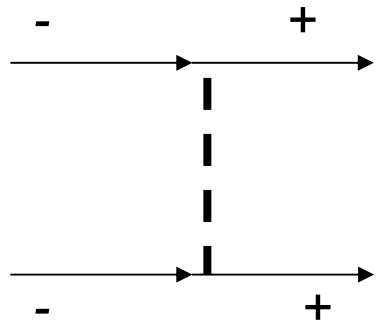
Spin degrees of freedom are split-off and gapless (free spin rotation phase θ).

Charge degrees of freedom can be gapful.

$$\Psi_{\pm} \sim \exp[\pm i\varphi/2]$$

Chiral phase $\varphi = \varphi(x,t)$ for electrons near $\pm K_F$:

Origine: Umklapp scattering (Dzyaloshinskii & Larkin, Luther & Emery).



$U \exp[i2\varphi]$: amplitude of the Umklapp scattering of electrons $(-K_F, -K_F) \rightarrow (+K_F, +K_F)$ is allowed here. Momentum deficit $4K_F$ is just compensated by the reciprocal lattice period

$$H = (\hbar/4\pi\gamma) [v_{\rho} (\partial_x \varphi)^2 + (\partial_t \varphi)^2 / v_{\rho}] - U \cos(2\varphi)$$

Hamiltonian degeneracy $\varphi \rightarrow \varphi + \pi$ originates current carriers:

$\pm\pi$ solitons with charges $\pm e$, energy Δ

(= holon = $4K_F$ CDW discommensuration = Wigner crystal vacancy)

Renormalization due to quantum fluctuations (Lee, Finkelstein, Wiegmann, ...) without interactions $\gamma=1$ and $v_{\rho}=v_F$

$\gamma < 1$: U is not renormalized to zero, common Mott insulator case

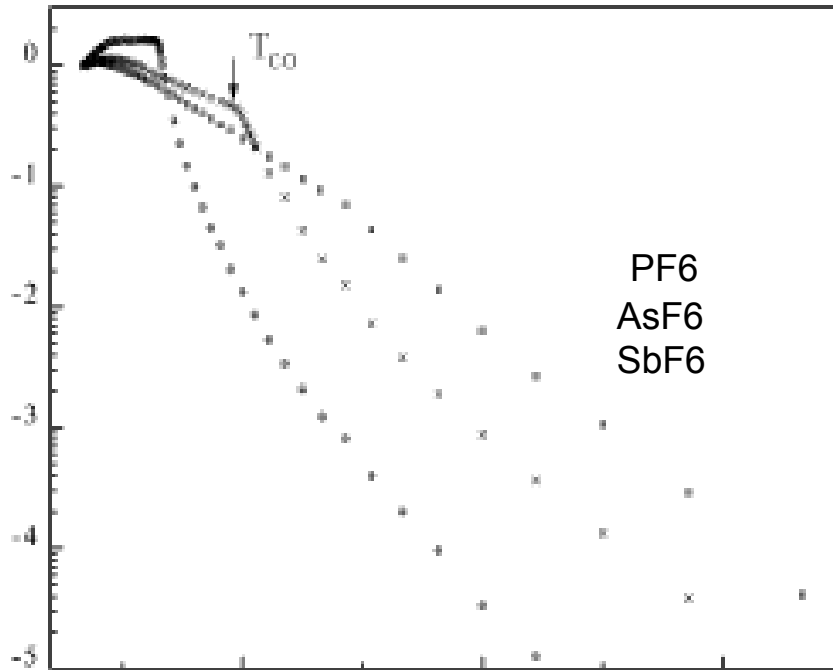
$\gamma < 1/2$: U can be spontaneously generated,
- new circumstances of charge ordering.

Facility to see Solitons - Purely 1D regime for electrons :
 $T_{FE} \approx 150K$ is 10 times above 3D electronic transitions.

Conductance G ,
normalized to RT :
Ahrenius plot $\text{Log } G(1/T)$.

Gaps for thermal activation Δ
range within 500-2000K.

after Nad and Monceau



Contrarily to normal semiconductors - no gap in spin susceptibility :
 $\chi(T)$ stays flat as for the parent metal.

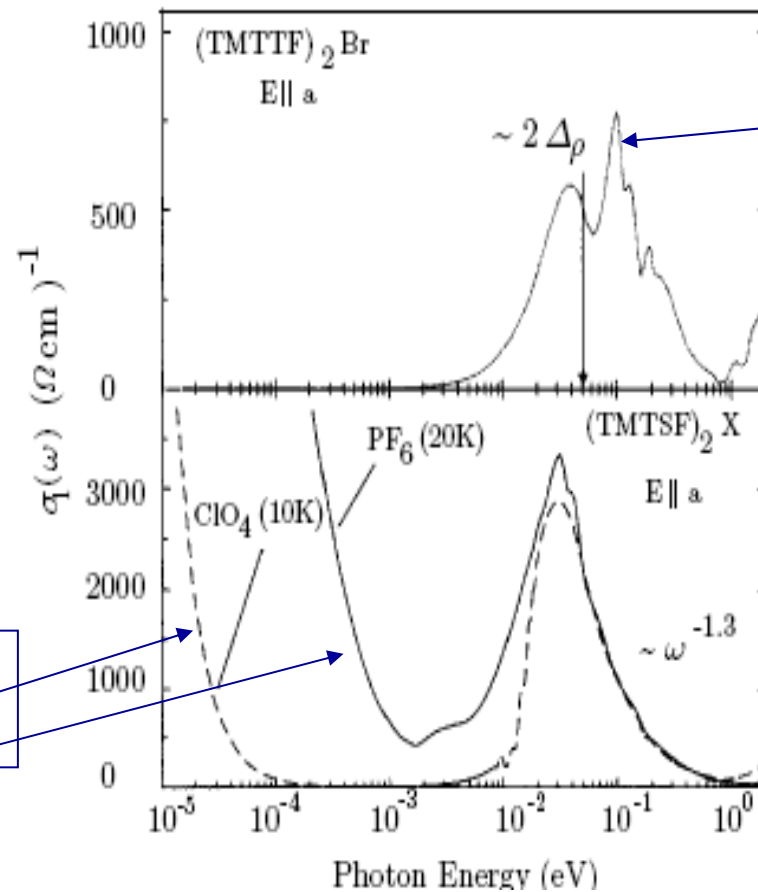
Clearlest case of conduction by charged spinless solitons -holons.

Beyond this talk - two other types of solitons observed in versions of same system :

1. Fractionally charged walls between opposite ferroelectric domains
2. Combined charge-spin solitons due to subsequent CDW transition of tetramerisation

Do we see the solitons in optics?
?

Seeing charge gap in optics:
Exploiting the data by
Digeorgi group, ETH



Optical Conductivity $\sigma(\omega) = \text{absorption}$

The peak due to either 2Δ of pairs of kinks production or E_g - optical absorption edge (exciton = bound kink+antikink)
Notice identity of static (TMTTF case)
and fluctuational (TMTSF case) Mott states

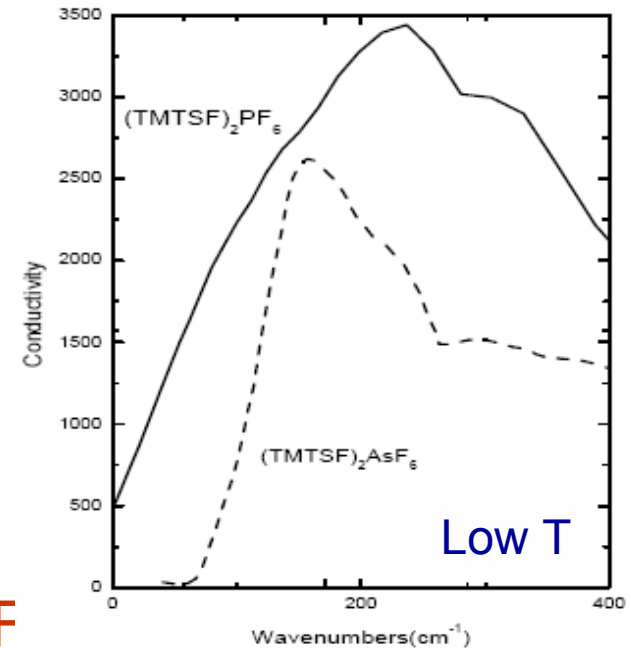
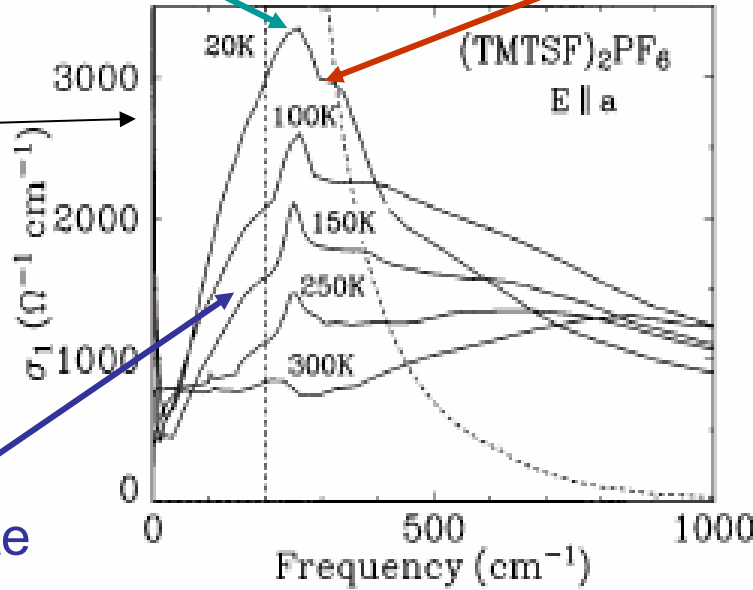
Do we see the solitons in optics ? Optical Conductivity, ETH group

$E_g = 2\Delta$ - pair of free kinks.

collective mode or exciton
= two kinks bound state

Drude peak –
It is a metal !

optically active
phonon of FE state



Illustrative interpretation of optics on TMTSF
in terms of firm expectations for CO/FE state in TMTTF's

Vocabulary :

TMTTF – compounds found in the Mott state, charge ordering is assured

TMTSF – metallic compounds, Mott and CO are present fluctuationally

SUMMARY

- Existence of solitons is proved experimentally in single- or bi-electronic processes of 1D regimes in quasi 1D materials.
- They feature self-trapping of electrons into midgap states and separation of spin and charge into spinons and holons, sometimes with their reconfinement at essentially different scales.
- Topologically unstable configurations are of particular importance allowing for direct transformation of electrons into solitons.
- Continuously broken symmetries allow for solitons to enter $D > 1$ world of long range ordered states: SC, ICDW, SDW.
- They take forms of amplitude kinks topologically bound to semi-vortices of gapless modes – half integer rotons
- These combined particles substitute for electrons certainly in quasi-1D systems – valid for both charge- and spin- gaped cases
- The description is extrapolatable to strongly correlated isotropic cases. Here it meets the picture of fragmented stripe phases

Some obligatory references – in theory only:

Brinkman & Rice

Dzyaloshinskii & Larkin

Fukuyama & Tanaka

Iordanskii and Rashba

Kirova

Kivelson&Auerbach

Kusmartsev

Luther & Emery

Matveenko

Mineev & Volovik

Nagaev

Schrieffer

Schultz

Shriman and Sigia

Wiegmann

Zaanen

etc., etc.