"What are topological excitations? … Think about local effects!"
- A.I. Larkin, in memoirs of V.L. Pokrovskii

Solitons in microscopic electronic processes in correlated systems
Recent experiments in light of Larkin inspirations

1960' s :   *Larkin – Ovchinnikov ; Fulde – Ferrell*
        Inhomogeneous state of spin – polarized superconductors

        Larkin – Khmelnitskii ; Nagaev
        Inhomogeneous state of doped ordered medias –
        Ferroelectric and magnetic conductors

1970's   *Larkin – Dzyaloshinskii ; Luther – Emery*
        Origin of the Mott AFM state in quasi 1D conductors

1980's   *Larkin – Ovchinnikov, Korshunov ;
        following Caldeira – Leggett*
        Instantons in dissipative quantum multi-state systems

Solitons in quasi one dimensional conductors *and beyond*

I. Ferroelectric Mott-Hubbard phase and charge ordering
in organic conductors – a firework of solitons

II. Solitons in dynamics and in the ground state of overlap tunnelling
junctions of incommensurate Charge Density Waves

III. Role of topological excitations in general strongly correlated systems

Solitons/instantons in electronic properties:
Born in theories of late 70's, Found in experiments of early 80's.
Why in 2000's ?
New conducting polymers,
New events in organic conductors,
New accesses to Charge Density Waves,
New interests in strongly correlated systems as semiconductors

Sources for this talk :
S.B. collaborations with joint experimental groups of
Grenoble (Monceau) and Moscow (F.Nad, Yu.Latyshev, et al),
Earlier theory work (S.B., N.Kirova, S.Matveenko, et al, etc.)

Solitons in general : propagating isolated profiles –
from the tsunami wave (Russel 1834) to
electron density droplets (Abanov&*Wiegmann - today*)

Solitons in our perspective of electronic systems :
Nonlinear self-localized excitations on top of a ground state with a
spontaneously broken symmetry: superconductivity, CDW,SDW,AFM
Solitons carry a charge or a spin – separately, even in fractions.
They bring associated spectral features (e.g. mid-gap states)
Instantons : related transient processes for creations of solitons or
their pairs, for conversion of electrons or excitons into solitons

Symmetry breaking :
Degenerate equivalent ground states.
Soliton= kink between them

Figure : energy as a function of configuration.
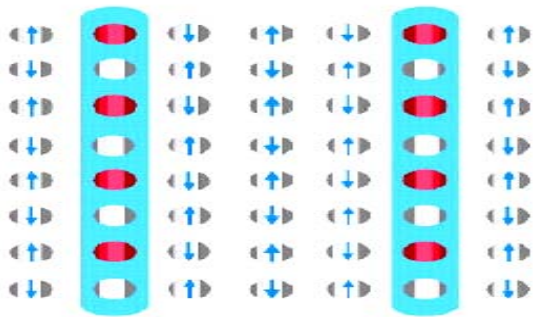two-fold degeneracy or
crossection of the axial-symmetry shape

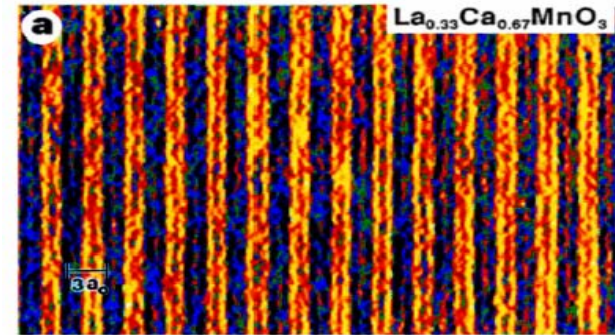-1   +1
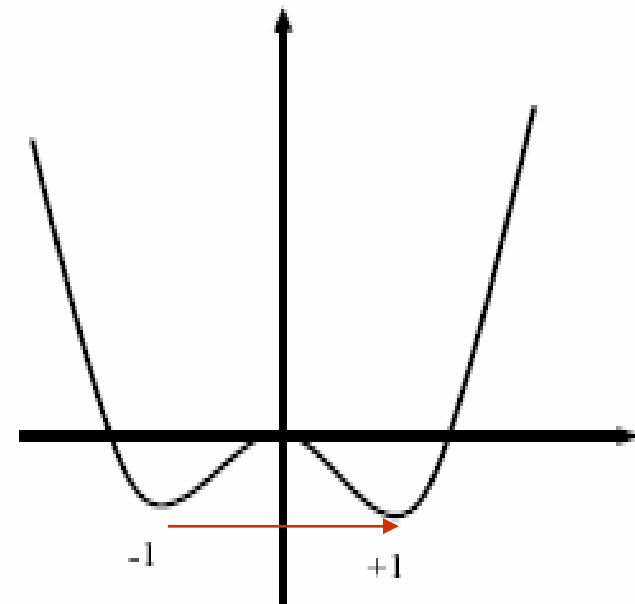
LOFF rout: from stripes to solitons

1D → quasi 1D → 2D,3D route to dopping of AFM insulator.
Aggregation of holes (extracted electrons) into stripes.

Left: scheme derived from neutron scattering experiments.
Right: *direct visualization via electron diffraction microscope.*

J.Orenstein et al Science 288, 468 (2000)

$La_{0.33}Ca_{0.67}MnO_3$

S.Mori et al Nature 392, 473 (1998)

Equivalence for spin-gap cases:
Fulde-Ferell-Larkin-Ovchinnikov FFLO phase in superconductors
Solitonic lattices in CDWs above the magnetic breakdown
Solitonic lattices in spin-Peierls GeCuO in HMF - Grenoble

Kink-roton complexes as nucleuses of melted macro structures: FF**LO** phase for superconductors or strips for doped AFMs.
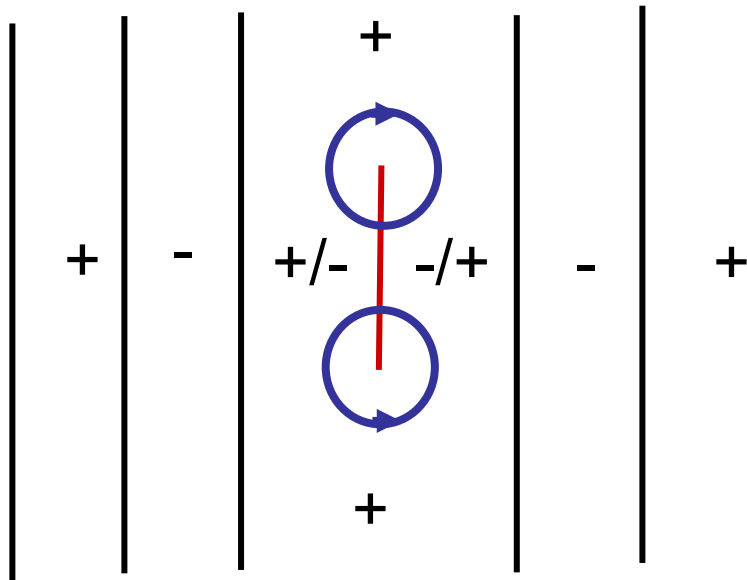
A defect embedded into the regular stripe structure (black lines).
+/- are the alternating signs of the order parameter amplitude.
Termination points of a finite segment (red color) of the zero line
must be encircled by semi-vortices of the π rotation (blue circles)
to resolve the signs conflict.
The minimal segment corresponds to the spin carrying kink.

Major and unifying observation :
combination of a discrete and continuous symmetries
Solitons are stable energetically but not topologically

Special significance: allowance for a direct transformation of
one electron into one soliton.
(Only 2$\rightarrow$2 are allowed for kinks in discrete symmetries)
Complex Order Parameter
O= A exp[iφ] ; **A** - amplitude , φ - phase
Ground State with an odd number of particles:
In 1D - *Amplitude Soliton* AS **O(x=-∞)** $\Leftrightarrow$ **- O(x=∞)**
via **A $\Leftrightarrow$-A** at arbitrary φ=cnst
Favorable in energy in comparison with an electron, but:
Prohibited to be created dynamically even in 1D
Prohibited to exist even stationary at D>1
RESOLUTION – Combined Symmetry :
A $\Leftrightarrow$-A combined with $\varphi\rightarrow\varphi+\pi$ – semi-vortex of phase rotation
compensates for the amplitude sign change

$H_{1D} \sim (\partial\theta)^2 - V\cos(2\theta)\} + (\partial\varphi)^2$

V - from the backward exchange scattering of electrons

In **1D** : Spinon as a soliton $\theta \rightarrow \theta+\pi$ hence **s=1/2**

+ Gapless charge sound in φ.

CDW order parameter $\sim \psi^\dagger_{+\uparrow} \psi_{-\uparrow} + \psi^\dagger_{+\downarrow} \psi_{-\downarrow} \sim$ **exp [iφ] cosθ**

- Its amplitude **cosθ** changes the sign along the allowed π soliton

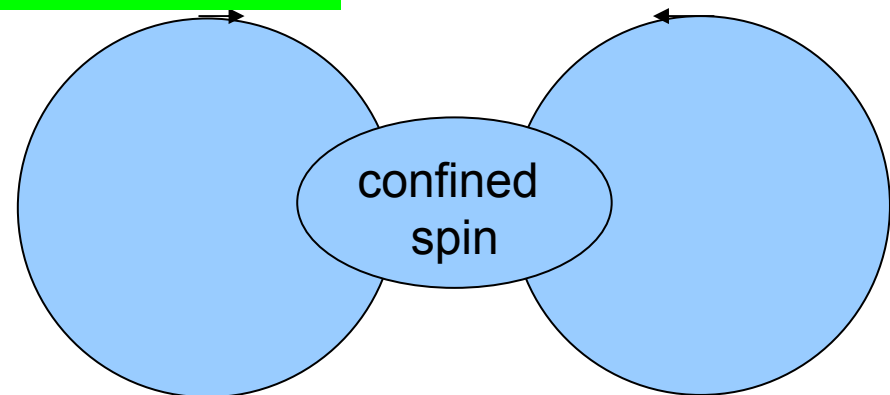At higher D : allowed mixed configuration

$\theta \rightarrow \theta+\pi$, **s=1/2** $\varphi \rightarrow \varphi+\pi$, **e=1**

↑spin soliton ↑   ↑charged wings↑

Spinon as a soliton +
semi-integer dislocation loop =
π - vortex of φ ≡ confined spin +
semi dislocation loop
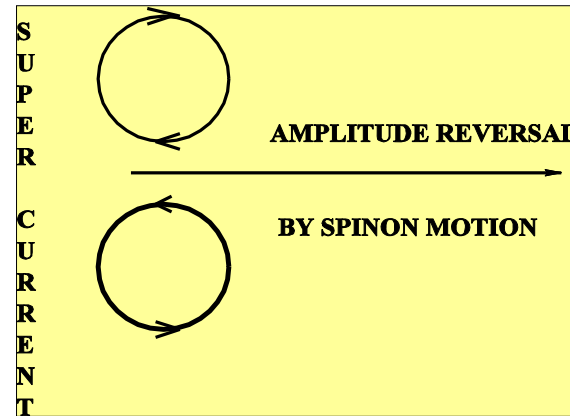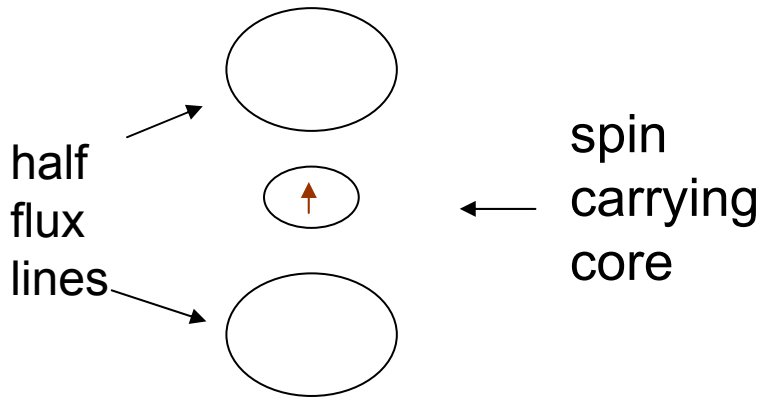
confined spin

Singlet Superconductivity: D=1 → D>1

$$\eta_{SC} \sim \psi_{+\uparrow}\,\psi_{-\downarrow} + \psi_{+\downarrow}\,\psi_{-\uparrow} \sim \exp[i\chi]\,\cos\theta$$

$\theta \rightarrow \theta+\pi$ s=1/2 $\chi \rightarrow \chi+\pi$

↑spin soliton ↑   ↑wings of supercurents↑

half flux lines

spin carrying core

S U P E R   C U R R E N T

AMPLITUDE REVERSAL

BY SPINON MOTION

Quasi 1d view : spinon as a π- Josephson junction in the superconducting wire   (*applications: Yakovenko et al*).

2D view : pair of π- vortices shares the common core
        bearing unpaired spin.

3D view : half-flux vortex stabilized by the confined spin.
Best view: nucleus of melted FFLO phase in spin-polarized SC

Half filled band with repulsion.
SDW rout to the doped Mott-Hubbard insulator.

$$H_{1D} \sim (\partial\varphi)^2 - U\cos(2\varphi)\} + (\partial\theta)^2$$

U - Umklapp amplitude
(*Dzyaloshinskii & Larkin ; Luther & Emery*).
φ - chiral phase of charge displacements
θ - chiral phase of spin rotations.
Degeneracy of the ground state:
$\quad \varphi \rightarrow \varphi + \pi =$ translation by one site

Excitations in 1D :

holon as a π soliton in φ, spin sound in θ
Higher D : A hole in the AFM environment.
Staggered magnetization $\equiv$ AFM=SDW order parameter:
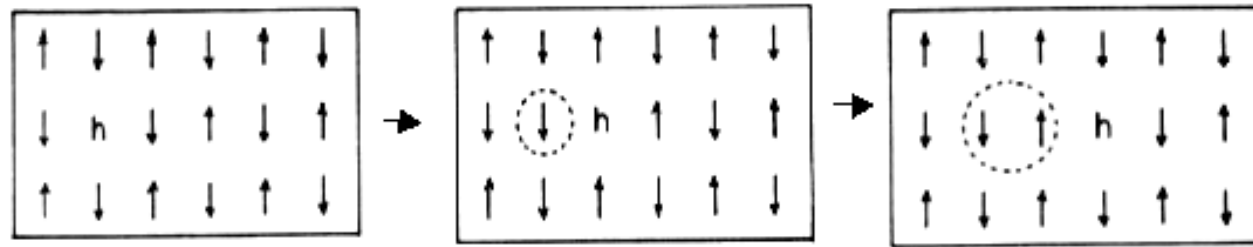**$O_{SDW} \sim \cos(\varphi) \exp\{\pm i(Qx+\theta)\}$**

To survive in D>1 :
The π soliton in φ cos $\varphi \rightarrow$ - cos φ
enforces a π rotation in θ to preserve O_{SDW}

Propagating hole as an amplitude soliton.
Its motion permutes AFM sublattices ↑,↓
creating a string of the reversed order parameter:
staggered magnetization.   It blocks the direct propagation.

Nagaev et al ,
Brinkman and Rice

Adding the semi-vorticity to the string end heals the permutation
thus allowing for propagation of the combined particle.

spin flipping

by hole motion

spin rotations
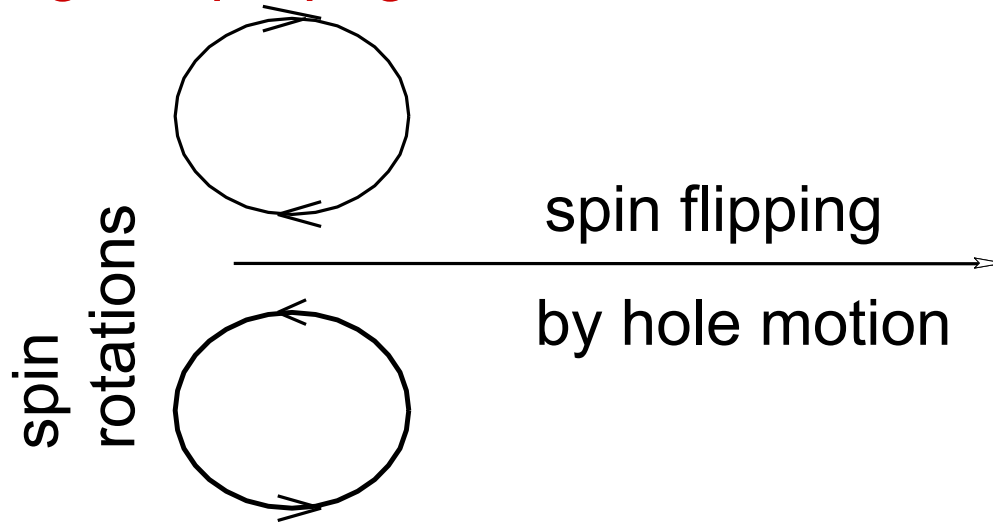
Alternative view:
Nucleus of the stripe phase or the minimal element of its melt.

Solitons and their arrays in incommensurate CDWs

Yu.Latyshev, P.Monceau, S. Brazovskii

Observation of Charge Density Wave Solitons in Overlapping Tunnel Junctions   Phys. Rev. Lett., **95**, 266402 (2005)

Subgap collective tunnelling and its staircase structure in charge density waves   Phys. Rev. Lett., **96**, 116402 (2006).

More on theory side :
S.B. and Serguei Matveenko, cond-mats/2000'
Instanton theory of the pseudogaps and of subgap transitions :
Applications to PES, ARPES, optics, tunneling.

Incommensurate Charge Density Wave – ICDW ~cos(Qx + φ)
Minimal view : uniaxial crystal of singlet electronic pairs,
defined on chains of the parent crystal.
Incommensurability allows for arbitrary ICDW displacements,
hence the complex order parameter Aexp(i φ)
- isomorphisme to superconductivity   (*Efetov and Larkin*)

(a)

● Nb, Ta, Zr
○ S, Se

(b)

○ Nb
○ Se
○ 0, b₀
● b₀/2

ρ(x)

ε(k)

2Δ

k_F
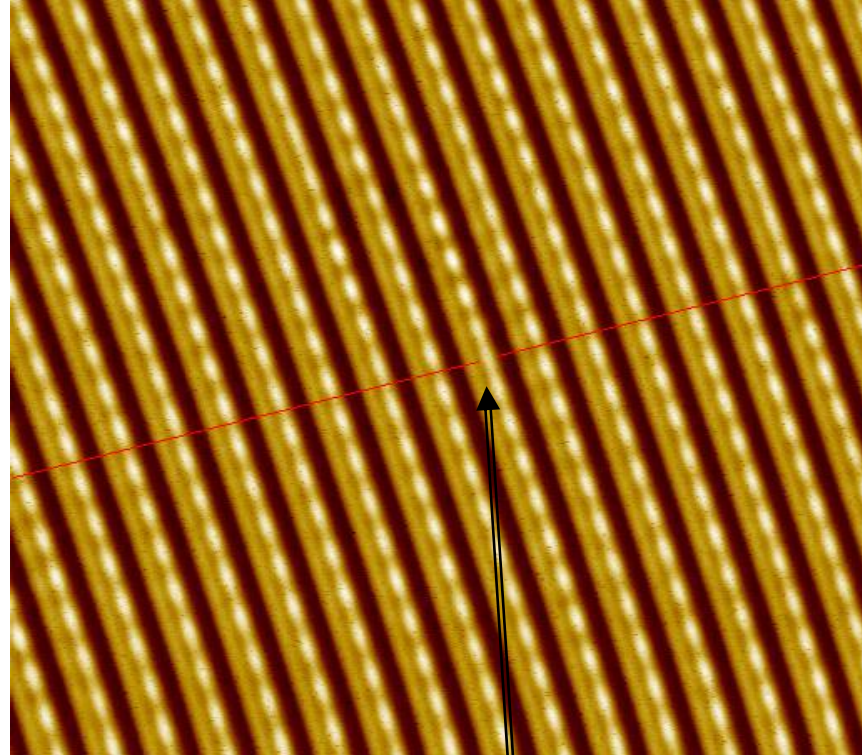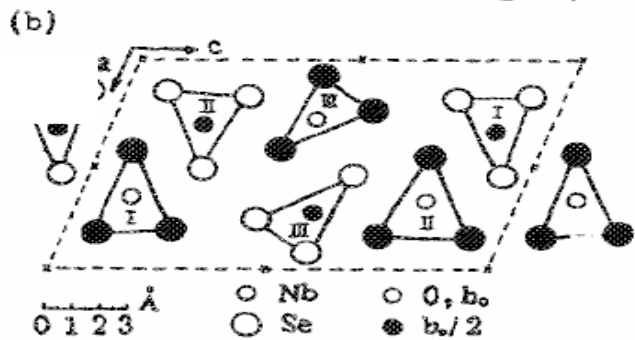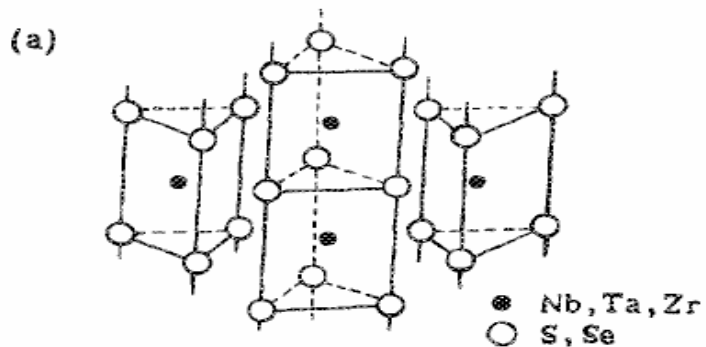
2e hole of the CDW crystal NbSe3
= 2π soliton = vortex pair =
result of the 2π phase slip
STM image, C. Brun and Z.Z. Wang
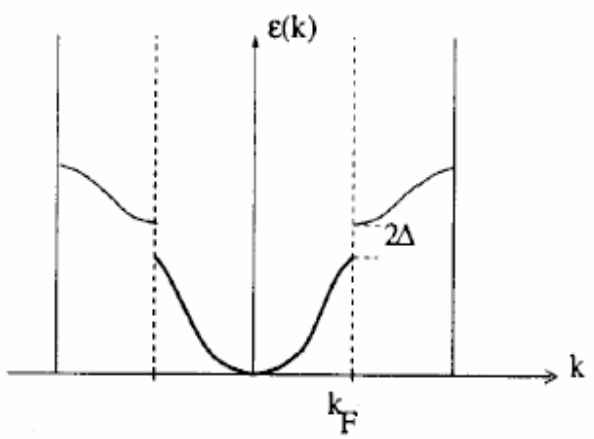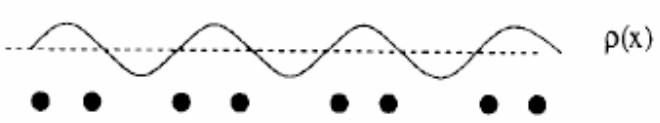
CDW period $\lambda = \pi / k_f$

density $\sim \cos(2k_f x + \varphi)$
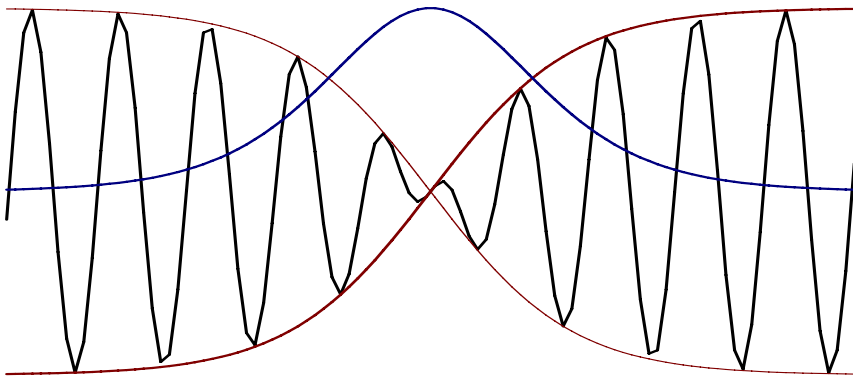
Gap opened at the Fermi level

What should happen if the singlet pair is broken
into spin ½ components ?

This creature appears in tunneling *(S.B., 3 decades anniversary):*
Amplipitude soliton with energy ≈2/3Δ ,
total charge 0, spin ½
This is the CDW realization of the SPINON

Oscillating electronic density,
Overlap soliton A(x),
Midgap state =spin distribution

Solitons as a result of electrons' selftrapping – next slide :

Microscopics of electrons conversion in ICDW:

Incommensurate CDW : $A\cos(Qx+\varphi)$ $Q=2K_f$
Order parameter : $\Delta \sim A\exp(i\varphi)$
Electronic states $\Psi = \Psi_+\exp(iK_fx+i\varphi/2) + \Psi_-\exp(-iK_fx-i\varphi/2)$

$$\begin{vmatrix} k-E & \Delta^* \\ \Delta & -k-E \end{vmatrix}\begin{pmatrix} \Psi_+ \\ \Psi_- \end{pmatrix} = 0 \quad , \quad k = K - K_f = -i\partial_x$$

Peierls-Frohlich, chiral Gross-Neveu models.
Spectra are related to the nonlinear Schroedinger equation for Δ:
Fateev, Novikov, Its, Krichever; Matveenko and S.B.

In equillibrium : $\Delta=\Delta_0=\text{const}, \ E=\pm(\Delta_0^2+k^2)^{1/2}$
Major interest: spectral flow between the two branches
of allowed states $E> \Delta_0$ and $E< -\Delta_0$ and the related
conversion of added particles to the extended ground state
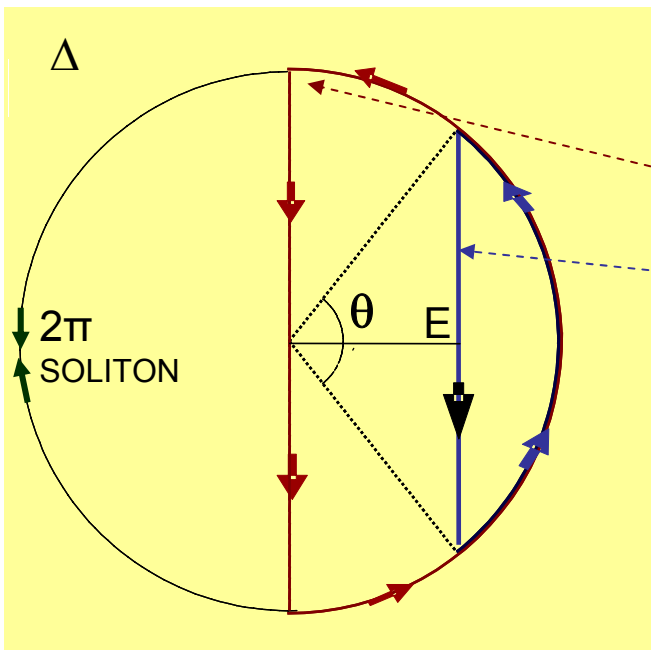
Soliton trajectories in the complex plane of the order parameter.
Red line: stable amplitude soliton.
Blue line: intermediate chordus soliton within chiral angle θ (black radial lines).
The value θ=100° is chosen which corresponds to the optimal configuration for the interchain tunnelling
S.Matveenko and S.B.
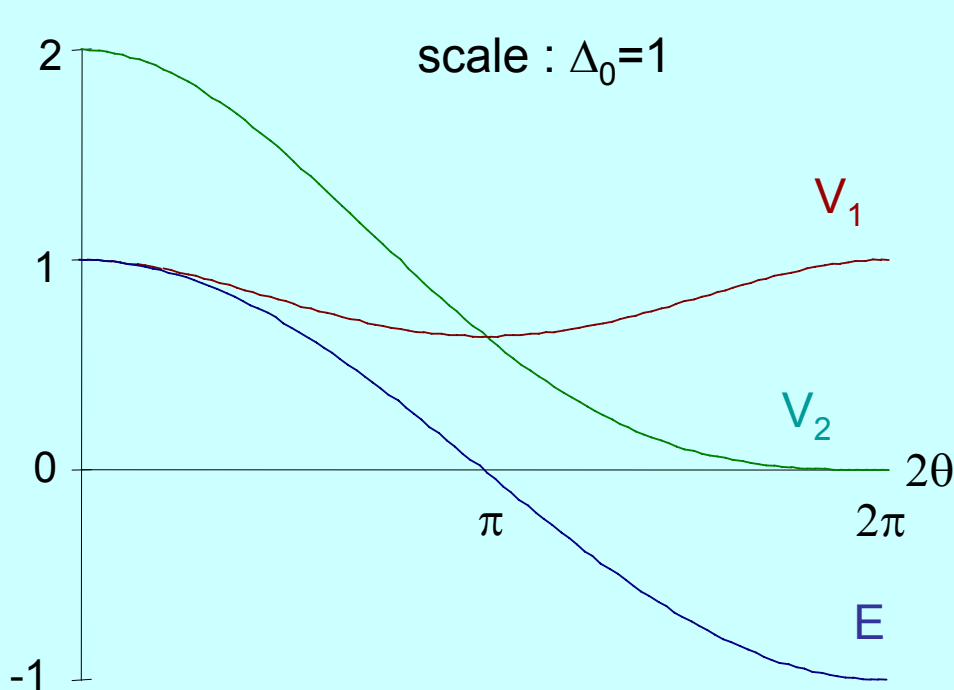
Selftrapping branches $V_n(\theta)$ for chordus solitons
with fillings n =1 and n=2,
Energy $E(\theta)$ of localized split-off state - Spectral flow between gap edges $\Delta_0 \rightarrow -\Delta_0$
No barrier for selftrapping in 1D !

Incommensurate CDWs in quasi 1D conductors.
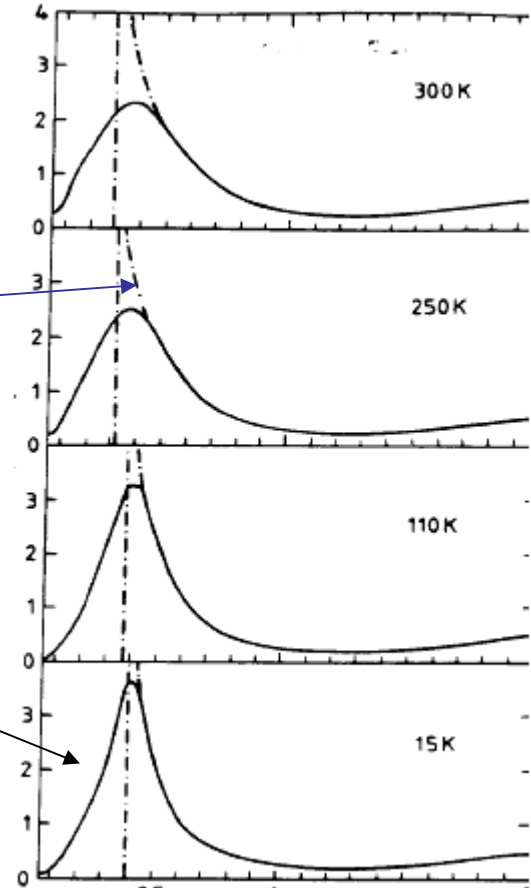Subgap transitions due to non-adiabatic quantum fluctuations;
Related pseudogap (*cf. cuprates*).
Already a long standing problem in optics :

Figure – DiGiorgi group, ETH
Optical absorption in Blue Bronze

Expected edge singularity at 2Δ

Subgap tail persistent at low T.
Quantum processes -
presence of excitations below 2Δ

Recovering a full and rich picture – New access by experiments of tunneling, particularly in overlap mesa-junctions
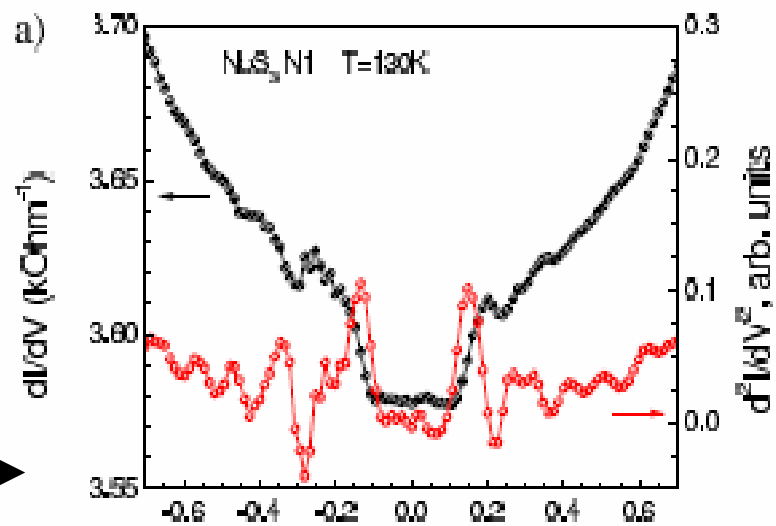*(Latyshev, Monceau, Sinchenko, …. S.B. – IRE&MIFI&ITP, Grenoble, Orsay)*

Direct observation of solitons and their arrays in tunneling on **NbSe3**

All feature scale with $\Delta(T)$!

creation of solitons at $2\Delta/\pi$: $E_s = 2\Delta/\pi$!

NbSe$_3$ stack N2

peak 2Δ for inter-gap creation of e-h pairs
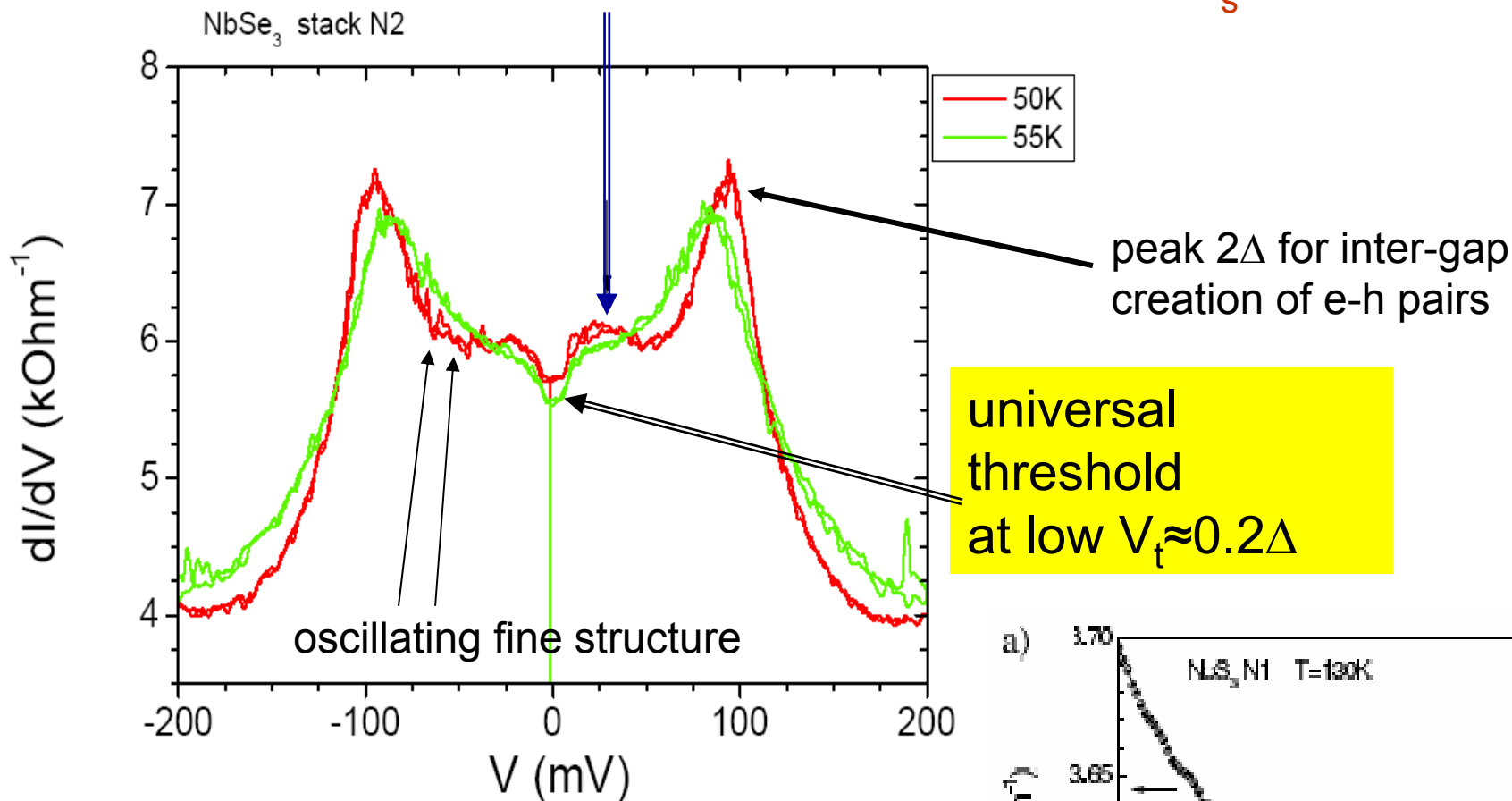
universal threshold at low $V_t \approx 0.2\Delta$

oscillating fine structure

fine structure is not a noise !
its interpretation :
sequential entering of dislocation lines
into the junction area.


a)   NbSe$_3$ N1   T=130K
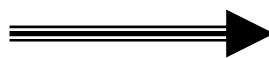
Fine structure within the magnifed threshold region.
Conductance *dI/dV* and its derivative d^2I/dV^2
Voltage *V* is normalized to the CDW gap.

Comparison of d^2I/dV^2 for
two voltage polarities for both CDWs,
   at T=130K and 50K;

the positive polarity at T=120K

Peaks interpretation :
sequential entering of dislocation lines into the junction area.
Critical voltage - DL entry energy, like Hc1 in superconductors.
(*S.B.-Matveenko 1989*)
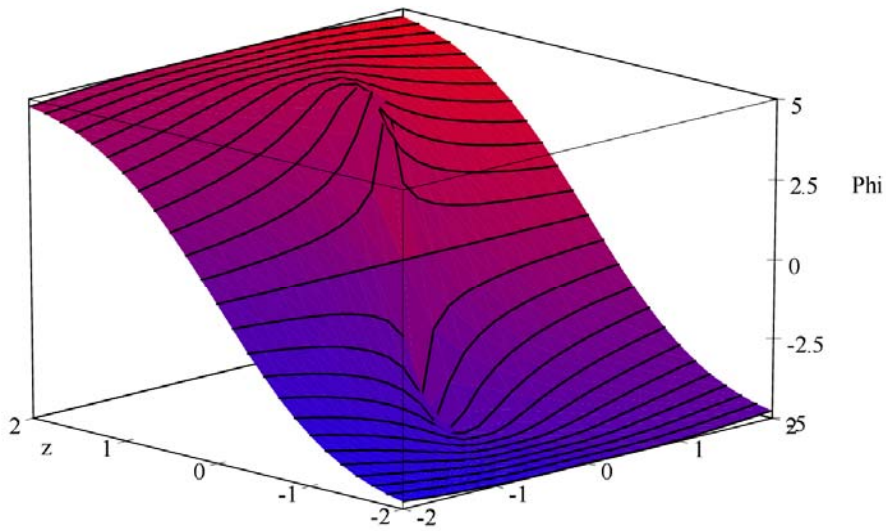
Potential distribution in a DL vicinity. Notice concentration of the potential Φ(x,z) drop, facilitating the tunneling.
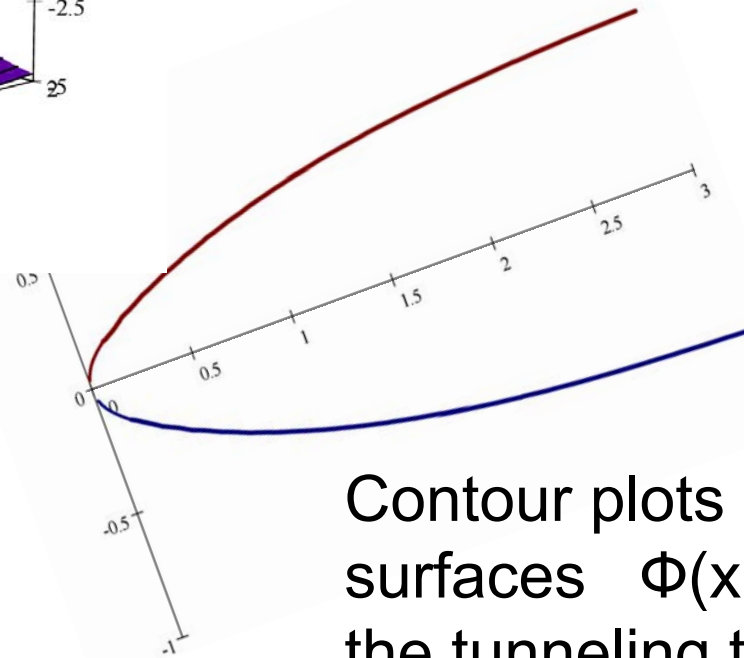
Contour plots ±z(x) for surfaces Φ(x,z)±Δ where the tunneling takes place.
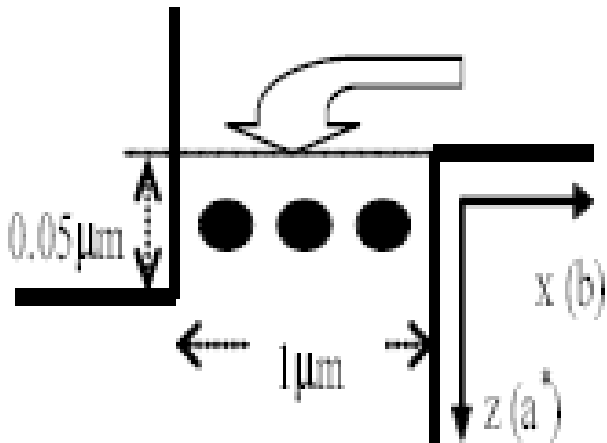
Junction scheme with crossections of dislocations

Pair of ±2π solitons is created by tunneling near the dislocation core, Interpretation: excitation of the dislocation line as a quantum string.

Phase mode action :   (*u*- phase velocity)

$$S_{snd} = \frac{v_f}{4\pi} \int\int dxdt((\partial_t\varphi/u)^2 + (\partial_x\varphi)^2)$$

The discontinuity is enforced by the Chordus soliton forming around $x_s=0$

Integrate out $\varphi(x,t)$ from $\exp(-S_{snd}[\varphi,\theta])$ - arrive for $\theta(t)$
at the typical action for the problem of quantum dissipation

$$S \sim \sum|\omega||\theta_\omega|^2 \qquad \dot{\theta} = \partial_t\theta$$

$$S_{snd}\{\theta] \approx -\frac{v_F/u}{2\pi^2}\int\int dt_{1,2}\dot{\theta}(t_1)\ln|(t_1 - t_2)|\dot{\theta}(t_2)$$

$$S_{snd} \approx (v_F/4u)\ln(uT/\xi_0)$$

AS edge $\Omega \geq W_s$ - the power law $I(\Omega) \propto \left(\frac{\Omega - W_s}{W_s}\right)^\beta, \; \beta = \frac{v_F}{4u}$

Summary for tunneling:
Specifics of strongly correlated electronic systems
inorganic CDW, organic semiconductors,
conjugated polymers, conducting oxides, etc…

Electronic processes, in junctions at least, are governed by solitons or more complex nonlinear configurations.

As proved by presented experiments, they can lead to:

- Conversion of a single electron into a spin solitons
- Conversion of electrons pair into the 2π phase slip
- Pair creation of solitons (tunneling and optics)
- Arrays of solitons aggregates

 – dislocation lines, walls of discommensurations –

reconstruct the junction state and provide
self-assembled micro-channels for tunneling;

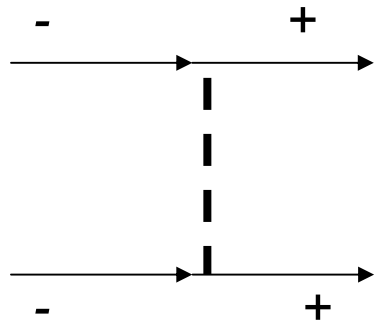1D Mott-Hubburd state. 1 electron per site i.e. the half filled band.

Spin degrees of freedom are split-off and gapless (free spin rotation phase θ).

Charge degrees of freedom can be gapful.

Chiral phase $\varphi=\varphi(x,t)$ for electrons near $+/-K_F$:

$$\Psi_\pm \sim \exp[\pm i\varphi/2]$$

Origine: Umklapp scattering (Dzyaloshinskii & Larkin, Luther & Emery).

$U\exp[i2\varphi]$: *amplitude of the Umklapp scattering of electrons* $(-K_F, -K_F) \rightarrow (+K_F, +K_F)$ is allowed here. Momentum deficit $4K_F$ is just compensated by the reciprocal lattice period

$$H= (\hbar/4\pi\gamma) [v_\rho (\partial_x\varphi)^2 + (\partial_t\varphi)^2 /v_\rho] - U\cos (2\varphi)$$

Hamiltonian degeneracy $\varphi \rightarrow \varphi+\pi$ originates current carriers:

$\pm\pi$ solitons with charges $\pm e$, energy Δ

(= holon = $4K_F$ CDW discommensuration = Wigner crystal vacancy)

Renormalization due to quantum fluctuations (Lee, Finkelstein, Wiegmann, …)
without interactions γ=1 *and* vρ=v$_F$

$\gamma<1$:   U is not renormalized to zero, common Mott insulator case

$\gamma<1/2$:   U can be spontaneously generated,
          - new circumstances of charge ordering.

Facility to see Solitons - Purely 1D regime for electrons :
$T_{FE} \approx 150K$ is 10 times above 3D electronic transitions.
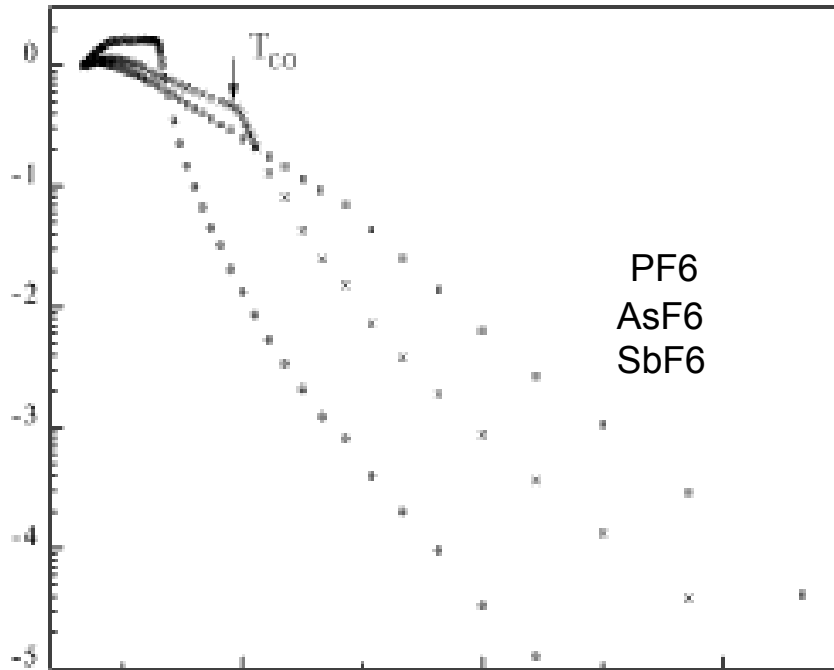
Conductance G ,
normalized to RT :
Ahrenius plot Log G(1/T).

Gaps for thermal activation Δ
range within 500-2000K.

after Nad and Monceau

Conrarily to normal semiconductors - no gap in spin susceptibility :
χ(T) stays flat as for the parent metal.
Clearlest case of conduction by charged spinless solitons -holons.
Beyond this talk - two other types of solitons observed in versions of same system :
1. Fractionally charged walls between opposite ferroelectric domains
2. Combined charge-spin solitons due to subsequent CDW transition of tetramerisation

Do we see the solitons in optics ?

Seeing charge gap in optics: Exploiting the data by Digeorgi group, ETH
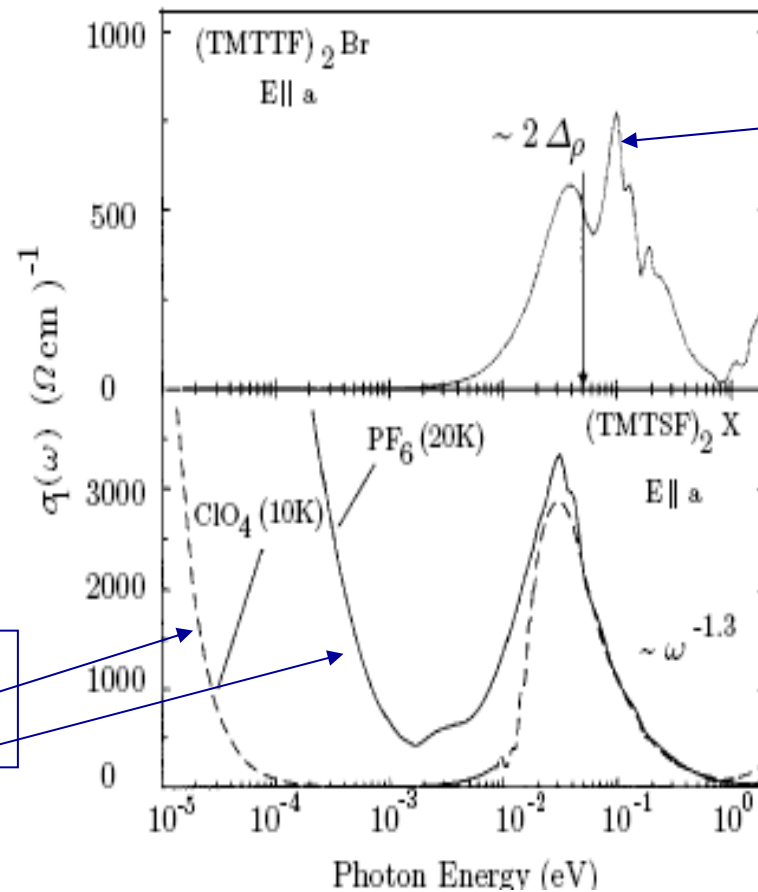
Parasite intra-molecular modes

Drude peak – It is a metal !

Optical Conductivity $\sigma(\omega)$ = absorption

The peak due to either 2Δ of pairs of kinks production or

E_g - optical absorption edge (exciton = bound kink+antikink)

Notice identity of static (TMTTF case)

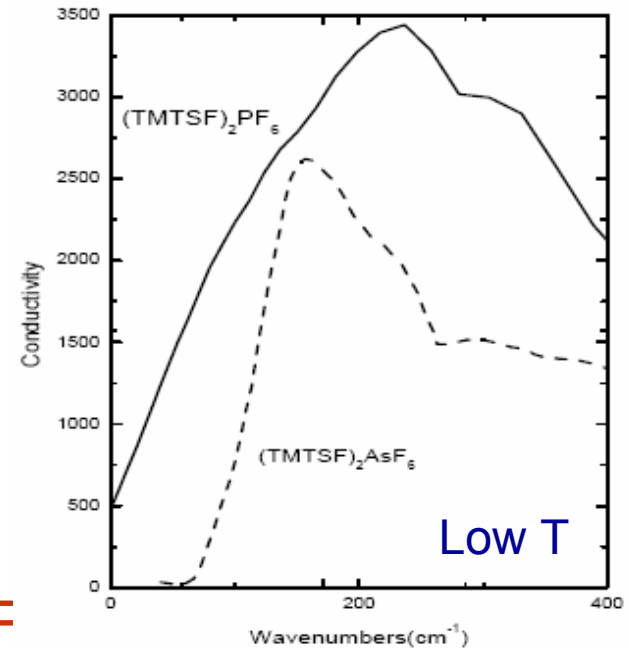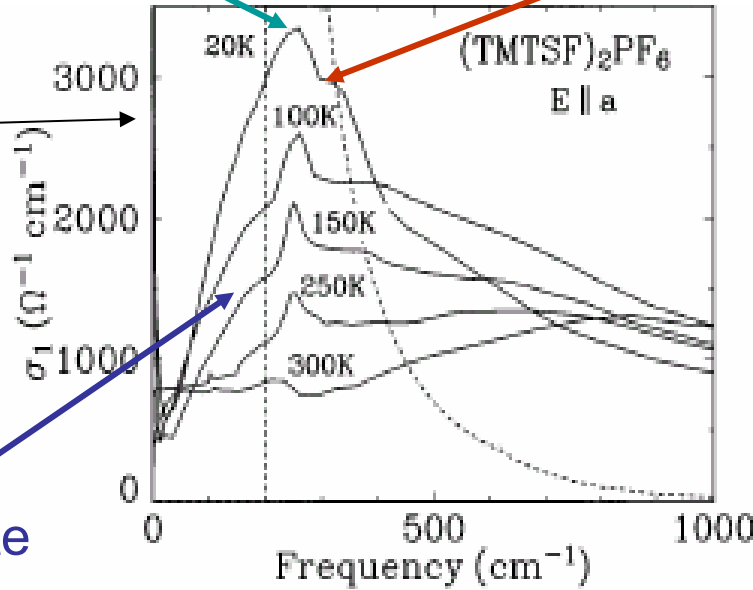and fluctuational (TMTSF case) Mott states

Do we see the solitons in optics ?

Optical Conductivity, ETH group

$E_g = 2\Delta$ - pair of free kinks.

collective mode or exciton
= two kinks bound state

Drude peak –
It is a metal !

optically active
phonon of FE state

$(TMTSF)_2PF_6$
$E \parallel a$

20K
100K
150K
250K
300K

σ $(\Omega^{-1} cm^{-1})$

3000
2000
1000
0

0   500   1000
Frequency (cm^{-1})

$(TMTSF)_2PF_6$
$(TMTSF)_2AsF_6$

Conductivity

3500
3000
2500
2000
1500
1000
500
0

0   200   400
Wavenumbers(cm^{-1})

Low T

Illustrative interpretation of optics on TMTSF
in terms of firm expectations for CO/FE state in TMTTF's

Vocabulary :
TMTTF – compounds found in the Mott state, charge ordering is assured
TMTSF – metallic compounds, Mott and CO are present fluctuationally

SUMMARY

- Existence of solitons is proved experimentally in single- or bi-electronic processes of 1D regimes in quasi 1D materials.

- They feature self-traping of electrons into midgap states and separation of spin and charge into spinons and holons, sometimes with their reconfinement at essentially different scales.

- Topologically unstable configurations are of particular importance allowing for direct transformation of electrons into solitons.

- Continuously broken symmetries allow for solitons to enter D>1 world of long range ordered states: SC, ICDW, SDW.

- They take forms of amplitude kinks topologically bound to semi-vortices of gapless modes – half integer rotons

- These combined particles substitute for electrons certainly in quasi-1D systems – valid for both charge- and spin- gaped cases

- The description is extrapolatable to strongly correlated isotropic cases. Here it meets the picture of fragmented stripe phases

Some obligatory references – in theory only:

Brinkman & Rice
Dzyaloshinskii & Larkin
Fukuyama & Tanaka
Iordanskii and Rashba
Kirova
Kivelson&Auerbach
Kusmartsev
Luther & Emery
Matveenko
Mineev & Volovik
Nagaev
Schrieffer
Schultz
Shriman and Sigia
Wiegmann
Zaanen
etc., etc.