Magnetoresistance of granular superconductors at low temperatures

Igor Beloborodov

Materials Science Division, Argonne
University of Chicago


Motivation:

New materials paradigm: Artificial Nanosolids

metals, semiconductors, superconductors, ferromagnets, hybrid nanostructures

Fundamental physical parameters for nanosolids

i) Nanograin sizes – electron confinement
ii) Coupling between grains – electron tunneling
iii) Electron interaction – Coulomb blockade
iv) Dimensionality – multiple tunneling pathways
Properties of a single grain

- single grain conductance

\[ g_0 = \frac{E_T}{\delta} \quad \text{- single grain conductance} \]

15 nm Al grain: \( \delta \approx 0.1 \text{ meV} \quad E_T \approx 10 \text{ meV} \)

\[ \tau = \frac{1}{E_T} = \frac{R^2}{D} \]
\[ \tau = \frac{1}{E_T} = \frac{R}{v_F} \]
Single superconducting grain

Focus on grain sizes > 7 nm

\[ \Delta > \delta \]  - condition for superconductivity in a single grain

\[ \Delta \sim \delta \]  - big fluctuations of \( \Delta \) destroy superconductivity

Anderson’59
Nanosolids characterized by two conductances

- grain conductance \( g_0 \)
- tunneling conductance \( g_T \)

- granular \( g_0 > g_T \)
- homogeneous \( g_0 \approx g_T \)
Array of superconducting grains

Assumptions: $\delta < t, \Delta, E_C < E_T$

- good metal
- small grains ($R < \xi$)

$R$ - single grain size
$\xi$ - coherence length for corresponding bulk sample
Model Hamiltonian

$\hat{H} = \hat{H}_{BCS} + \hat{H}_T$

$\hat{H}_{BCS} = \hat{H}_0 - |\lambda| \sum_{i,k,k'} a_{i,k}^+ a_{i,-k}^+ a_{i,-k'} a_{i,k'}$

free e + disorder

e-e attraction

$\hat{H}_T = \sum_{ij} t_{ij} a_i^+ a_j + c.c.$

tunneling
Effect of magnetic field on superconductivity

**Orbital**

bulk - $B_C^{or} \xi^2 \approx \Phi_0$

grain - $B_C^{or} \xi R \approx \Phi_0$

**Zeeman**

Zeeman effect:

$B_C^{or} \approx \frac{R_C}{R}$

Orbital effect:

$\mu_B B_C^{Z} \approx \Delta$

**Critical Sample Size**

$R_C = \Phi_0 \sqrt{D/\Delta}$

- **Orbital** $R > R_C$
- **Zeeman** $R < R_C$

Beloborodov et al., PRB 61, 9145 (2000)
Magnetoresistance of granular superconductors

Gerber et al’ 97

- grain size 120 Å
- B up to 17 T

\[ T_C \approx 1.58K \quad (B = 0) \]

FIG. 4. Normalized resistances of sample 1 (open circles) and sample 2 (triangles) as a function of applied magnetic field measured at \( T = 0.3 \) K. Inset: magnetoresistance of sample 3 at \( T = 2 \) K normalized at \( H = 160 \) kOe.
Magnetoresistance of granular superconductors

\[ \sigma = \sigma_1 + \sigma_2 + \sigma_3 \]

- **Single electron tunneling**
  \[ \sigma_1 \sim \nu_i(\varepsilon)\nu_j(\varepsilon) \]
  
  - No interaction
  - Disorder & interaction

- **Virtual Cooper pair tunneling**
  \[ \sigma_{2,3} \sim T^2 \] at \( T \ll T_C \)
  
  - Cooper pairs localized
  - No current contribution

Suppression of conductivity
Magnetoresistance of granular superconductors

1) Superconducting fluctuations at low temperatures and high magnetic fields lead to density of states suppression
2) All Cooper pairs are localized

\[ R \]

\[ B \]

conductority reduction

\[ T \ll T_c \]

Beloborodov, Efetov, Larkin
PRB 61, 9145 (2000)
Insulating state of granular superconductors

EXPERIMENT:
Gantmakher et al.’00
Sambandamurthy al.’04
Baturina et al.’04
Steiner et al.’05
Paalanen et al.’92

SUPERCONDUCTOR

METAL

perturbation theory

INSULATOR

$R$ vs. $B$

$\sim (3 - 5) T$
Insulating state of granular superconductors

Gantmakher, et al. ‘2000

In-O, perpendicular field

Experiment: strong grains coupling

G. Sambandamurthy et al. ‘2004
Superconductor – Insulator transition in granular metals

Efetov ’ 80

$E_c \sim E_J$ - SI transition

Insulating state possible for $E_c > E_J$

experiment: $g > 1$, $E_c \rightarrow E_c^{\text{eff}} \sim \Delta/g$

$E_J \sim g \Delta \gg E_c^{\text{eff}} \rightarrow$ superconducting state

We need different model!
Insulating state of granular superconductors

- grains of slightly different sizes
- magnetic field \( \rightarrow \) change relative fraction of superconducting and normal grains

In 2D exist concentrations of sites where simultaneously neither black nor white sites percolate
Insulating state: theoretical description

\[ g_{ns} >> g_{nn} , g_{ss} \]

\[ S = S_{ci} + S_{ns} \]

\[ \text{for } g_{ns} >> 1 \]

\[ S \sim \int d\tau \int d^2q E_c(q) | \Phi_q(\tau) |^2 \]

\[ E_c(q) = (E_c^{-1} + B[1 - E_q])^{-1} \]

Due to magnetic field:

\[ B \sim g / \Delta_0, \]

\[ E_q = \frac{1}{2} \sum_a \cos qa \]
Insulating state of granular superconductors

**gap in electron spectrum**

\[ \Delta \sim \left( \frac{\Delta_0}{g} \right) \ln \left( \frac{gE_C}{\Delta_0} \right) \]

\( \Delta_0 \) - superconducting gap at \( B=0 \)

\( g \) - tunneling conductance, \( E_C \) - charging energy

**Conductivity**

\[ \sigma \sim \exp \left( -\frac{\Delta}{T} \right) \]

**What is the applicability of this result?**

Stability of insulating state

with respect to formation of normal state
Electron tunneling via virtual state

![Diagram showing electron tunneling via virtual state]

small for $g < \sqrt{\Delta_0 / \delta}$

with respect to formation of superconducting state
Cooper pair tunneling

![Diagram showing Cooper pair tunneling]

small for $g < (\Delta_0 / \delta)^{1/3}$

Insulating state is stable for: $g < (\Delta_0 / \delta)^{1/3}$
Summary

For a magnetic field $B$, a metal-insulator-superconductor (MIS) transition occurs.

Magnetic field $B$

 Perturbation theory

$R$ vs. $B$

Superconductor

Insulator

Metal

$\sigma \sim \exp\left(-\frac{\Delta}{T}\right)$

$\Delta \sim \left(\frac{\Delta_0}{g}\right) \ln\left(\frac{gE_C}{\Delta_0}\right)$

Stable for:

$g < \left(\frac{\Delta_0}{\delta}\right)^{\frac{1}{3}}$